GENETICS

A PERIODICAL RECORD OF INVESTIGATIONS BEARING
ON HEREDITY AND VARIATION

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GENETICS

A PERIODICAL RECORD OF INVESTIGATIONS BEARING ON HEREDITY AND VARIATION

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CORRIGENDA

Page 15, table 1, column 4, for "158" read "138".

Page 16, line 7, for "1146" read "1246".

Page 30, table 16, heading, for "w"" read "w".

Page 42, table 34 *A*, pedigree 1320, under heading w^{e}vf s, for "— —" read "4 7".

Page 43, table 34 *C*, pedigree 1348, under sf, for "49" read "39".

Page 43, table 34 *D*, pedigree 1425, under w^{e}v, for "55" read "5".

Page 43, table 34 *D*, pedigree 1428, under sf, for "13" read "14".

Page 47, table 37 (*B*), pedigree 895, under v, for "7" read "1".

Page 65, section (8), omit sentence beginning "If *r*, *t*, and *s* are diverse", etc. In the next sentence, for "the next following generation", read "any following generation"; and omit the last paragraph of this section, beginning, "The proportions for succeeding generations," etc.

Page 70, last line of section (28), for "$\frac{91}{144}$" read "$\frac{85}{144}$".

Page 84, line 11, for "dominant" read "dominants".

Page 87, line 9, limiting value of *a*—, for "⅔" read "⅓".

Page 105, first line of footnote, for "this" read "these".

Page 107, line 11, for "cream" read "cream *a*"; for "whiting" read "dark and whiting"; and for "fawn" read "cream *b*".

Page 129, table 63, column 3, total, for "176" read "178".

Page 130, table 66, pedigree 1155, under v, for "36" read "16".

Page 182, second column of table 7, for "96" read "90".

Page 183, last column of table 8, the broken figure is a "7".

Page 196, line 12, for "Orman." read "Organ.".

Page 289, lines 28-29, for "red-and whites", read "red-and-whites".

Page 354, after name of author insert address, *"Columbia University, New York City"*.

Page 365, line 2, for "JANSSEN'S" read "JANSSENS'S".

Page 372, in legend of figure 2, for "pigment" read "areas".

Page 378, ninth line below table 1, for "*Y* chromosome" read "Y chromosome".

Page 394, last line, for "7.815 ± 0.122", read "6.282 ± 0.112".

Jhsvs

Sit nomen benedictum

Jacques Menvet

FRONTISPIECE—GREGOR MENDEL

JOHANN MENDEL was born July 22, 1822, of German peasant parentage, in Heinzendorf near Odrau, Austrian Silesia. He was educated in the gymnasium in Troppau, a year in an Academy at Olmütz, and later for three years (1851-1853) in the University of Vienna. He became a novitiate in the House of St. Thomas, or "Königinkloster," at Brünn, and in 1846 was admitted to the Augustinian Order, as "PATER GREGOR". After completing his studies at Vienna MENDEL became a teacher of natural science in the Staatsoberrealschule in Brünn, and began, in the gardens of the Monastery, his now celebrated investigations on hybridization in garden peas. His analysis of the problems of hybridization on the basis of single unit-characters constitutes his fundamental contribution to scientific methodology. Besides his extensive studies with peas MENDEL conducted genetical experiments with the hawkweeds (Hieracium) and with bees, and also produced several new varieties of fruits and flowers. The results of his work with peas were reported before the NATURAL HISTORY SOCIETY of Brünn in February and March 1865, and were published in the MEMOIRS of that society in 1866. They were also made known to the renowned botanist, CARL NÄGELI, through an extended correspondence covering the years 1866-1873, but remained practically unnoticed by other biologists until 1900, when they were brought to light almost simultaneously by DE VRIES, CORRENS and TSCHERMAK.

MENDEL was also greatly interested in climatology, and kept for many years a careful record of climatological data and of the occurrence of sun-spots.

In 1868 MENDEL was made administrative head of the Monastery and his numerous new duties brought his scientific studies to an abrupt close. After a long and strenuous career, much of which was embittered by a losing fight with the civil authorities over what he considered an unjust taxation of the properties in his charge, he died of chronic nephritis on January 6, 1884, scientifically unknown and apparently unconscious of the real importance which his scientific contributions must one day assume.

The portrait here reproduced is a photographic copy of an oil painting which now hangs in the parlor of the Monastery of which GREGOR MENDEL was for fifteen years the Abbot. Several futile attempts have been made by the Managing Editor to learn something of the history of this painting. Its reproduction here has been made possible by a special gift for this purpose, from a friend of GENETICS.

FOREWORD

The need of a journal in America for the publication of the longer and more detailed accounts of genetical investigations, with adequate illustrations, has been felt for some years. American papers in this field have been in the past distributed among a number of different journals, several of which are published in Europe, and this has been a serious inconvenience to American geneticists. It is hoped that the publication of GENETICS will obviate the necessity for so large a dependence upon the forbearance and the generosity of European journals, and that it will serve to concentrate and thus to make more generally available the bulk of American work. While the chief advantages from the establishment of GENETICS may be expected, for geographical reasons, to accrue to American investigators, contributions from the geneticists of other nations will be equally acceptable.

The eager interest with which modern discoveries in genetics have been followed, not only by all biologists, but also by those interested in eugenics and in the practical problems of plant and animal improvement, should assure a hearty welcome to a journal that devotes itself to the encouragement of fundamental investigations in genetics. The cordiality of this welcome has been foreshadowed by the subscriptions which have been pledged in advance by about 270 individuals and institutions who have by this means enrolled themselves among the founders of this journal. Without the support given by these friends of genetical research, the plan must have been abandoned, and the Editors desire to express their appreciation of the assistance thus given.

Scientific journals are generally published at a financial loss or are prevented by financial limitations from fully attaining the ideals of their Editors and Friends. The publication of such a journal as GENETICS

involves considerable sacrifice, not only on the part of those charged with its editorial management, but also on the part of the Publisher who does his best to produce a technically creditable journal and who assumes the financial risk. In order that this journal may become an adequate medium for the dissemination and preservation of the results of genetical research, the subscription list must be still further increased. The Editors and Publisher venture to hope, therefore, that all of those who have assisted in the founding of this journal will not only continue their subscriptions, but that they will also aid in enlarging the list of its supporters, in order that the income may produce a periodical worthy of the body of investigators who will find through its pages the means of communicating their discoveries. To these investigators and to those interested in the results of their investigations this periodical is dedicated.

THE EDITORS.

NON-DISJUNCTION AS PROOF OF THE CHROMOSOME THEORY OF HEREDITY[1]

CALVIN B. BRIDGES
Columbia University, New York City

[Received October 21, 1915]

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I

INTRODUCTION

There has been a long series of observations and experiments which has led more and more definitely to the conclusion that the chromosomes are the bearers of the hereditary materials.[2] It was observed that male and female contribute equally to the inheritance of the offspring, and yet the contribution of the male consists of little more than a nucleus. That

[1] Contribution from the Zoological Laboratory of Columbia University.

[2] For a fuller discussion of the steps in the accumulation of this evidence see "The mechanism of Mendelian heredity" by MORGAN, STURTEVANT, MULLER, and BRIDGES.

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I

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inheritance is a function of the nucleus rather than of the cytoplasm is shown by many embryological and cytological facts. Attention was next narrowed to the chromatin and chromosomes. Embryological experiments showed that the chromosomes are qualitatively different and that a full complement is essential to normal development. The increasing cytological knowledge of mitotic division and of gametogenesis made it clear that the chromosomes were qualified to serve as the material basis of heredity.

The next advance was the result of the exact knowledge of heredity which Mendelian analysis furnished, and of the coupling of experimental genetics with cytological investigation. It was shown that the genes for characters and the chromosomes have the same *method* of distribution. More recently cases have arisen in which genes and chromosomes have the *same* distribution. The final step has been to demonstrate the *identity of distribution between specific genes and specific chromosomes* in such a way that the argument of "the-cell-as-a-whole" cannot be applied, and in such a way that the chromosomes must be regarded as the means and not the consequence of the inheritance of characters and of sex. The experimental and cytological evidence in the case of non-disjunction furnishes such a demonstration.

An account of the discovery of non-disjunction and of its effect upon sex-linked inheritance was published (BRIDGES 1913 b) in the JOURNAL OF EXPERIMENTAL ZOOLOGY for November 1913. A further account in the form of a summary was given in SCIENCE, July 17, 1914 (BRIDGES 1914).

The work on non-disjunction started from exceptions in certain experiments which I was carrying out with Prof. T. H. MORGAN (MORGAN and BRIDGES 1913). The work was continued while I was assisting Dr. MORGAN, and my most sincere thanks are due for the opportunity and the encouragement which he offered. The frequent consultations and the constant association with Dr. A. H. STURTEVANT, Dr. H. J. MULLER, and other workers in the laboratory have brought out possibilities that would otherwise have been overlooked.

A brief statement concerning sex determination, normal sex-linked inheritance, etc., which form the background of these experiments may first be made.

THE SEX CHROMOSOMES AND SEX

The female of *Drosophila ampelophila* has a pair of sex chromosomes (X chromosomes) and three pairs of autosomes. The medium sized

straight chromosomes shown by the diagram to the left in figure 1 are identified as the pair of X chromosomes.

FIGURE 1.—Diagram showing the characteristic pairing, size relations, and shapes of the chromosomes of *Drosophila ampelophila*. In the male an X and a Y chromosome correspond to the X pair of the female. The relative lengths of the chromosomes of this diagram are based upon averages of the measurements of the figures in plate 1; on the basis of X = 100 the length of each long autosome is 159, of each small autosome 12, of the whole Y 112, of the long arm of the Y 71, and of the short arm 41.

The chromosomes of the male have presented serious difficulties in spite of the very great amount of study devoted to them. Miss STEVENS (1908 a) made a prolonged study involving the dissection and examination of some two thousand individuals. She described the three pairs of autosomes of the male as the same as those in the female, and the sex chromosomes as *"a clearly unequal pair"*.[3] The cytological work which I have done on males and on females having an extra Y shows that X is somewhat shorter than Y, that it is typically straight, and attached to the spindle fiber by its end. On the other hand, the Y chromosome (see figure 1 to the right) is somewhat longer than the X, is attached not by its end, but sub-terminally, and typically has the shape of a J or of a V with one of the arms shorter than the other. Drosophila is therefore a member of the group of forms in which the male produces two kinds of sperm, half with an X and half with a Y chromosome. All

[3] Miss STEVENS supposed the longer of these two chromosomes to be the chromosome present in duplex in the female, and that a distinct "X" piece constituted the middle of this long heterochromosome, so that at the time at which the first paper on non-disjunction was written (BRIDGES 1913 b), it had become the general belief that the constitution of the male was XO and of the female XX. The X chromosome was supposed to be attached to an autosome, as in Ascaris.

the eggs of the female are alike, each carrying an X after the polar bodies have been given off. The fertilization of an X egg by an X sperm results in an XX individual which develops into a female; the fertilization of an X egg by a Y sperm results in an XY individual which develops into a male.

NORMAL SEX-LINKED INHERITANCE

There are now about fifty sex-linked mutations known in Drosophila, and the data collected in their investigation are the most extensive known in experimental breeding. The arrangement of the sex-linked genes in a linear series and the establishment of the relative distances between the loci are based upon over half a million flies.[4] The work on non-disjunction deals directly with the best known of these sex-linked characters, and therefore rests upon a very firm Mendelian foundation.

The inheritance of a recessive sex-linked character may be illustrated by the cross of a vermilion female by a wild type (red-eyed) male (see figure 2). The sons are vermilion-eyed like the mother, and the daughters are wild type like the father. This criss-cross inheritance is explained by the theory that the genes for the sex-linked characters are carried by the X chromosomes. As shown in figure 2, the son derives his single X chromosome from his mother, and shows vermilion eye color because the gene for vermilion was carried by that chromosome. The Y chromosome from the father does not affect the visible characters of the son in any way. Thus a male always shows by its characters what genes are carried by its X chromosome. The daughter receives from the mother an X carrying the vermilion gene, but since the vermilion is recessive to the unmutated gene (red) carried by the X which she receives from the father, she will be wild type (red).

THE CHARACTERS USED AND THEIR LINKAGE RELATIONS

When vermilion is crossed to sable (a sex-linked recessive body-color) there are more vermilion grandsons and sable grandsons than grandsons that are both vermilion and sable or neither (i.e. wild type). If the gene for vermilion was carried by the maternal X and the gene for sable by the paternal X, the vermilion sable and wild type grandsons must have been produced through a process which may be called crossing over. The simplest way in which this crossing over may be assumed to take place

[4] Carnegie publication No. 237 by MORGAN and BRIDGES gives the most recent account of the sex-linked characters and of the linkage data which had been obtained prior to June 1914.

is shown in figure 3. The two X chromosomes come together and twist about each other, with homologous regions of the two chromosomes lying side by side. The genetic evidence requires that the chromosomes break across and then reunite in such a way that each of the chromo-

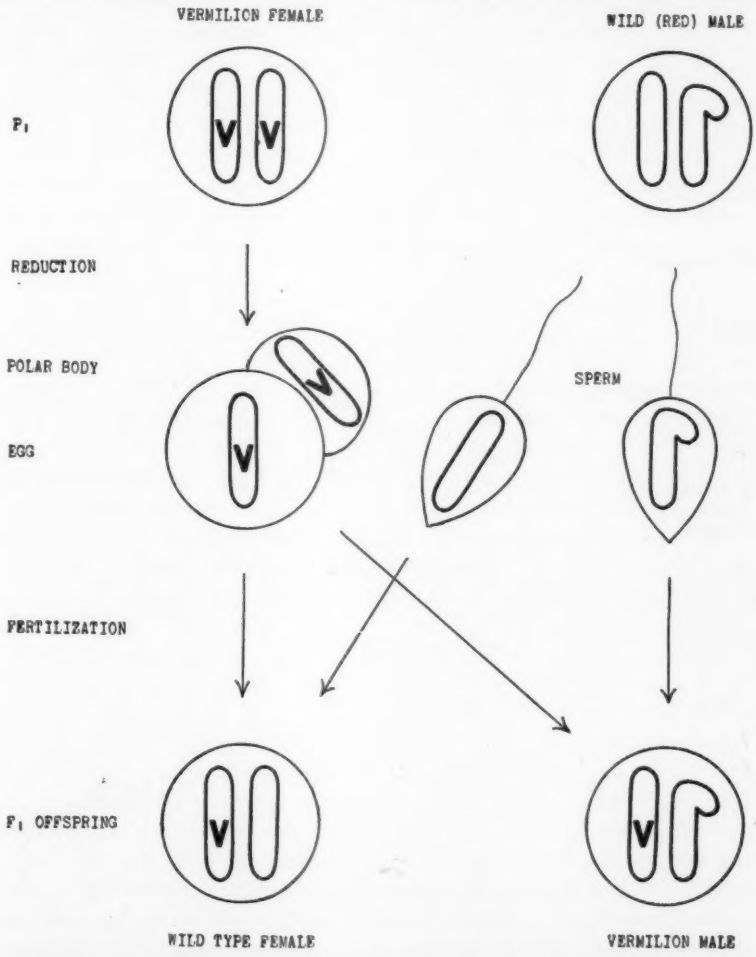


FIGURE 2.—The relations of the sex chromosomes to sex production and to the inheritance of the sex-linked character vermilion eye-color. The X chromosomes are represented as straight rods and the Y as J-shaped. The females with v in only one of the two X's do not show the recessive vermilion character.

somes is composed of an original piece joined to the succeeding piece of its mate. Crossing over may be supposed to occur at any point along the chromosome. The right side of the diagram of figure 3 illustrates such a crossing over occurring between the loci for vermilion and sable.

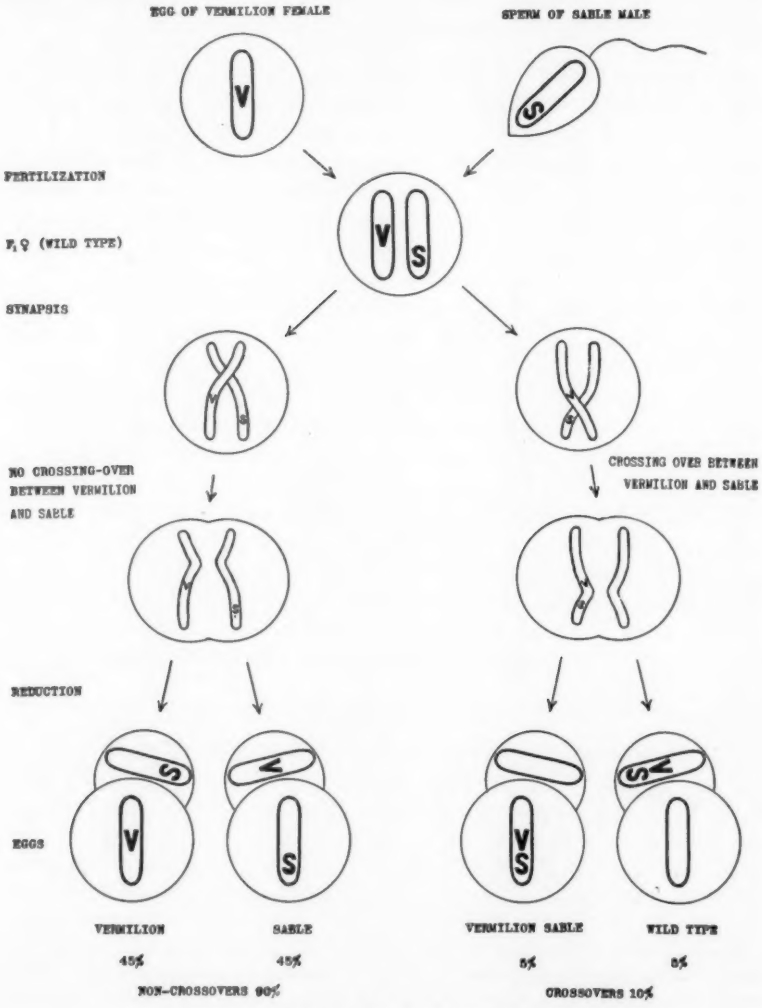


FIGURE 3.—Diagram to show crossing over.

Only ten percent of all the offspring are found to have resulted from crossing over between vermilion and sable. It is known from certain experiments that in about forty percent of cases there is no crossing over whatever between the two X chromosomes. Of the sixty percent of cases in which crossing over occurs, ten are between vermilion and sable. The remaining fifty percent of cases represent crossing over at one or more points in some other region of the X; these ineffective crossovers plus the forty percent of absolute non-crossovers give ninety percent of non-crossovers with respect to vermilion and sable. The percentage in which such a doubly heterozygous female produces gametes which are crossovers between vermilion and sable is tested by breeding her to the double recessive, vermilion sable, male. Each of the resulting flies shows characters corresponding directly to the genes of the maternal gamete from which it came.

In the case of vermilion and sable ten percent of the offspring are crossovers; in the case of vermilion and miniature (miniature is a sex-linked wing-character) only 3.1 percent are crossovers. We interpret this difference to mean that the gene for miniature lies in the chromosome much nearer to vermilion than does sable—that miniature lies at a distance of 3.1 units from vermilion while sable lies 10 units from vermilion. On this basis the positions of the genes for the most important of the sex-linked characters have been mapped out. As much of this map as is needed for the account that follows is given in figure 4.

Of the characters shown on the map, the only one which is dominant to the wild type is bar (eye-shape). Yellow, tan, and sable are body colors; miniature is a wing-character; forked refers to a scraggly branching of the spines of the head and thorax; white, eosin, cherry, and vermilion are eye-colors. White, eosin, and cherry occupy the same locus in the X chromosome, that is, they are allelomorphs of one another. A female which carries white in one X and eosin in the other has an eye-color which is intermediate between white and eosin and which is called a white-eosin compound. Eosin eye-color is markedly dimorphic, the males are a pinkish yellow, while the females are a slightly yellowish pink. When a fly shows two or more non-allelomorphic characters, the names are written from left to right in the order of their position from the zero end of the map. Thus, a yellow forked fly shows both yellow body-color and forked spines. Eosin vermilion, a double recessive form in which both characters are effects upon the eye-color, is a pale cream color (darker in the female than in the male) very easily separated from both eosin and vermilion.

The system of symbols used in the diagrams and table headings is as follows. The gene for a recessive mutant character is represented by a lower-case letter, as t for tan and v for vermilion. The symbols for eosin and cherry, which are allelomorphs of white, are w^e and w^c. The symbol for the dominant mutant character, bar, is B′. The symbols for the allelomorphs of the mutant genes are the converse letters, as b′ (not-bar), but are now usually omitted. The same letters that are used to

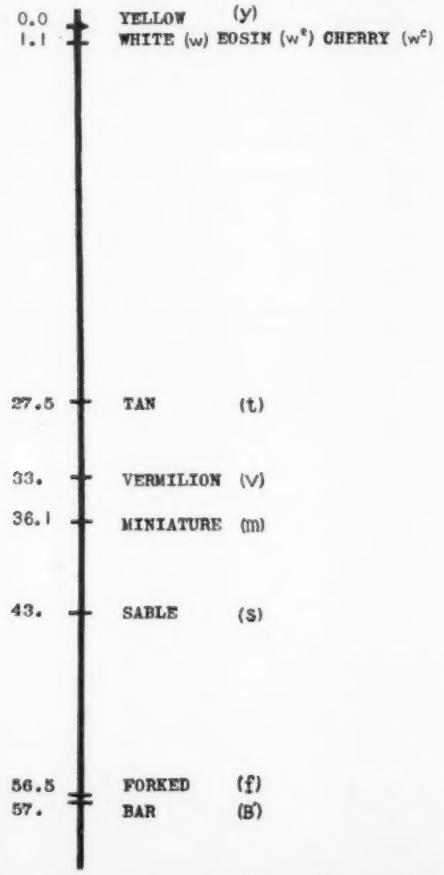


FIGURE 4.—Map of the linear arrangement and relative spacing of the genes for sex-linked characters. The letters, used both as the symbols for the genes and as abbreviations of the names of the characters, are included in parentheses.

represent the genes are also used as symbols of the names of the somatic characters shown by the flies. The symbol + is read "wild type" and indicates that the flies classified under this heading show no mutant characters.

PRIMARY NON-DISJUNCTION IN THE FEMALE

Ordinarily, as in diagram 2, in a cross to a male with the dominant character all the sons and none of the daughters show the recessive sex-linked characters of the mother. Similarly, all the daughters and none of the sons show the dominant sex-linked characters of the father. The peculiarity of non-disjunction is that sometimes a female transcends these rules and produces a daughter like herself (a matroclinous daughter) or a son like the father (a patroclinous son), while the rest of the offspring are perfectly regular, showing the expected criss-cross inheritance. Such exceptions, produced by a normal XX female, may be called primary.

The production of primary exceptions by a normal XX female may be supposed to result from an aberrant reduction division at which *the two X chromosomes fail to disjoin from each other. In consequence both remain in the egg or both pass out into the polar body.* In the former case the egg will be left with two X chromosomes and in the latter case with no X.

If the genes for sex-linked characters are carried by the X chromosomes, then each of the X chromosomes of the XX egg of a vermilion female will carry the gene for vermilion. The fertilization of such XX and zero eggs by the X and by the Y spermatozoa of a wild male will result in four new types of zygotes, as shown in figure 5.

(1) The XX egg fertilized by the X sperm gives an XXX zygote which might be expected to develop into a female. No females of this class have been found, and it is certain that they die.

(2) The fertilization of the XX egg by the Y sperm gives rise to a female having an extra Y chromosome (XXY). Since both of the X chromosomes came from the vermilion-eyed mother, this daughter must be a vermilion *matroclinous exception.*

(3) The fertilization of the zero egg by the X sperm gives rise to a male which has no Y chromosome (XO), and whose X coming from his red-eyed father brings in the red gene which makes the son a *patroclinous exception.* These XO males are viable but are completely sterile.

(4) The zero egg by the Y sperm gives a zygote (OY) which is not viable.

Perhaps the cause of the initial aberrant reduction which constitutes primary non-disjunction is a mechanical entanglement (an incomplete untwisting from a strepsinema stage) of the two X chromosomes, resulting in a delayed reduction. In such cases the formation of the cell boundaries would catch the lagging X's and include them in one or the other cell, and perhaps very often (as in certain nematodes) would pre-

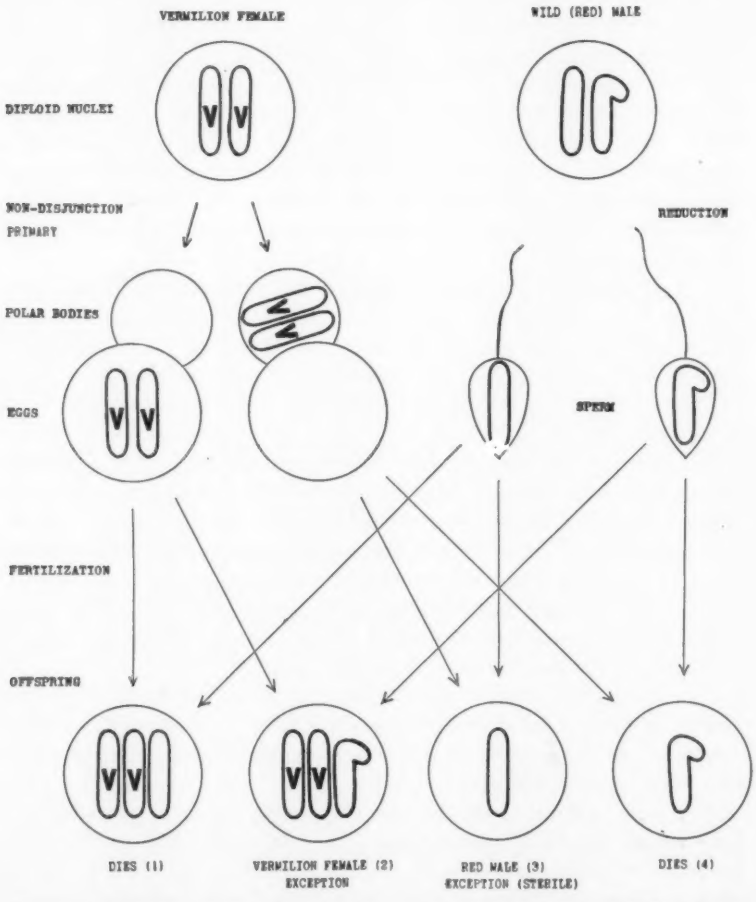


FIGURE 5.—Diagram of the production by a vermilion-eyed female of XX and zero eggs through *primary* non-disjunction, and the progeny resulting from the fertilization of such eggs by the sperm of a red-eyed male.

vent their leaving the middle of the spindle to join either daughter nucleus. If such an occurrence were common there should be more zero than XX eggs and consequently more primary exceptions should be males than females.

In studying primary non-disjunction we are dependent on what material chance offers, since we know of no means of controlling the process. It is equally as likely that an egg produced by primary non-disjunction will become a non-viable zygote (XXX and OY) as that it shall be viable (XXY and XO). For this reason it is impossible to detect half of those cases of primary non-disjunction which really occur. The XO male is viable and should offer an interesting field for further work, but—he is sterile. The direct opening offered for further work is through the matroclinous XXY daughter, which is perfectly fertile and which produces further exceptions which we may call secondary.

PRIMARY NON-DISJUNCTION IN THE MALE

If primary non-disjunction occurred in the male, XY and zero sperm would be formed, but the zygotes from them would not differ in their sex-linked characters from regular offspring, so that such an occurrence could not be detected immediately. However, the XY sperm would give rise to XXY daughters, and these in turn would produce secondary exceptions which could be observed.

Primary non-disjunction has been actually seen to occur in the male of Metapodius. WILSON found three spermatocytes in which X followed Y to one pole at the reduction division (WILSON 1909).

SECONDARY NON-DISJUNCTION IN THE FEMALE

It has been shown that matroclinous daughters of the constitution XXY may arise as the result of primary non-disjunction. The results from the outcrossing of several matroclinous daughters to males having other sex-linked characters were given in 1913. Of unusual interest was the appearance in F_1 of about four[5] percent of further exceptions (secondary exceptions). That is, about four percent of the daughters were like the mother and four percent of the sons were like the father. The remaining sons and daughters were of the kinds expected.

The explanation given at first for the fact that exceptional daughters inherit from their mother the power of producing exceptions, was that each X of the exceptional female carried a gene which caused these

[5] In previous papers the percentage of exceptions has been given roughly as five. The mean of all data now on hand is 4.3 percent of exceptions (see page 16).

chromosomes to undergo reduction abnormally in a small percentage of cases. Since these chromosomes descend directly to their exceptional daughters, they would transmit to those daughters the same gene and consequently the same power of producing exceptions.

Later work has provided data which can not be explained by appealing to the action of a gene in the X chromosome, and which prove that these secondary exceptions are due to the presence of the extra Y. In an XXY female there are three homologous sex chromosomes, between any two of which synapsis may occur, that is, synapsis may be of the XX or the XY type (homo- and heterosynapsis). In only about sixteen percent of cases (see page 17) does heterosynapsis occur, while about 84 percent of cases are homosynaptic and the Y remains unsynapsed. At the reduction division the two chromosomes that have synapsed, disjoin, one going to each pole, and the free chromosome goes to one pole, as often with the one as with the other of the disjoined chromosomes. Thus, after heterosynapsis the reduction divisions are of two kinds, the XX-Y and the X-XY types. Half the eggs that come from the XX-Y type of reduction are XX and the other half are Y. For the X-XY type the eggs are X and XY, as many of one kind as of the other. After homosynapsis all the reductions are X-XY. As a result of reduction of these two types there are four classes of eggs—two of which, X and XY, are composite and large (46 percent), and two of which, XX and Y are of single origin and small (4 percent). If these eggs are produced by a vermilion-eyed female, both of whose X chromosomes carry the recessive gene for vermilion, then the eight classes of zygotes shown in figure 6 will result upon fertilization by a wild male, which produces X and Y sperm.

The XX eggs fertilized by X sperm give XXX individuals (figure 6, 1) which are unable to live.

The XX eggs fertilized by the Y sperm give individuals (5) which are exact duplicates of their mother in their sex chromosomes, and like her are females each containing an extra Y chromosome. Since the gene for vermilion is carried by the X chromosome, these females have vermilion eyes and hence are matroclinous exceptions. Since they have not received an X from their father, they can neither show nor transmit his sex-linked characters. If in the mother the presence of the extra Y led to the production of secondary exceptions, then these XXY daughters should also give exceptions, and this is in fact the case.

The Y eggs fertilized by the X sperm give males (2). These males have received their X from their father and they show his sex-linked

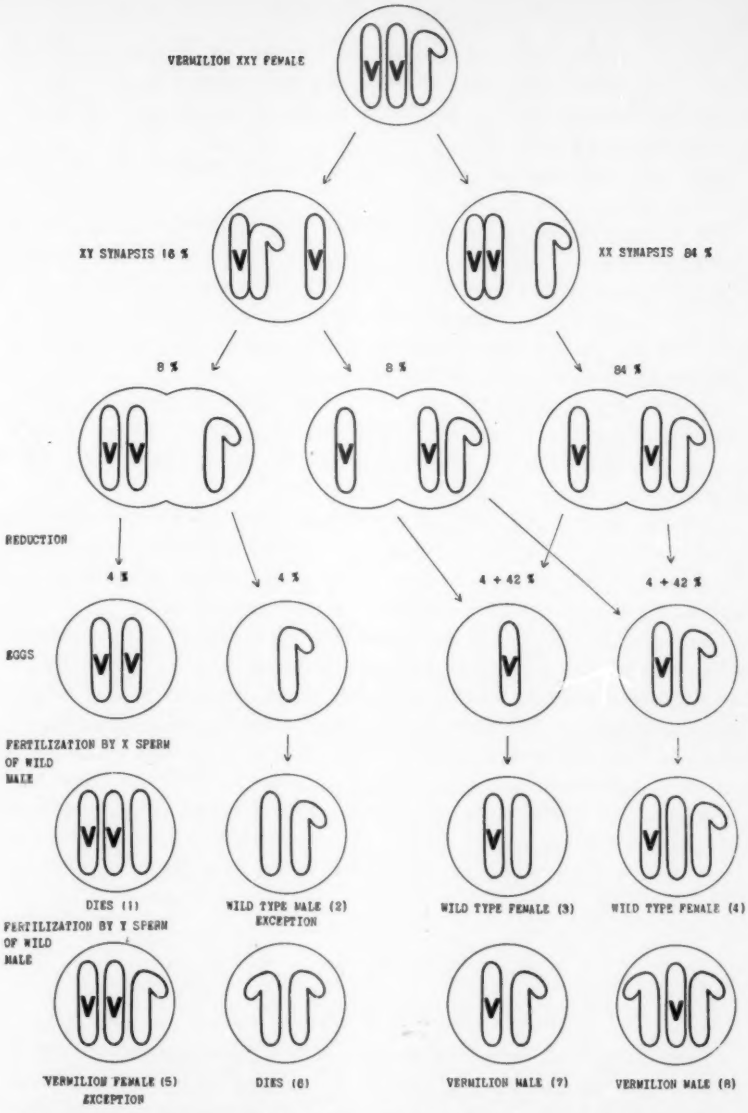


FIGURE 6.—Secondary non-disjunction in the female. Diagram showing the constitution of an exceptional vermilion female, the two types of synapsis, reduction, and the four classes of eggs produced. Each kind of egg may be fertilized by either of the two (X and Y) kinds of sperm of the wild male, giving the eight classes of zygotes shown.

characters, that is, they are patroclinous exceptions. Since in chromosome constitution (XY) these males are not different from ordinary males, they should have no power of producing exceptions. This has been shown to be the case.

The Y eggs fertilized by Y sperm give YY individuals (6) which are unable to live.

The X eggs by X sperm give regular XX females (3), and by Y sperm give regular XY males (7). Neither of these two classes is able to produce secondary exceptions or to transmit non-disjunction.

The XY eggs by X sperm give XXY females (4) *which, because of the extra Y, possess the power of producing secondary exceptions, though they themselves are not exceptions.*

The XY eggs by Y sperm give XYY males (8). These males do not give rise to genetic exceptions in F_1, but they endow some of their daughters with an extra Y (XY sperm and X egg) which enables these daughters to produce secondary exceptions.

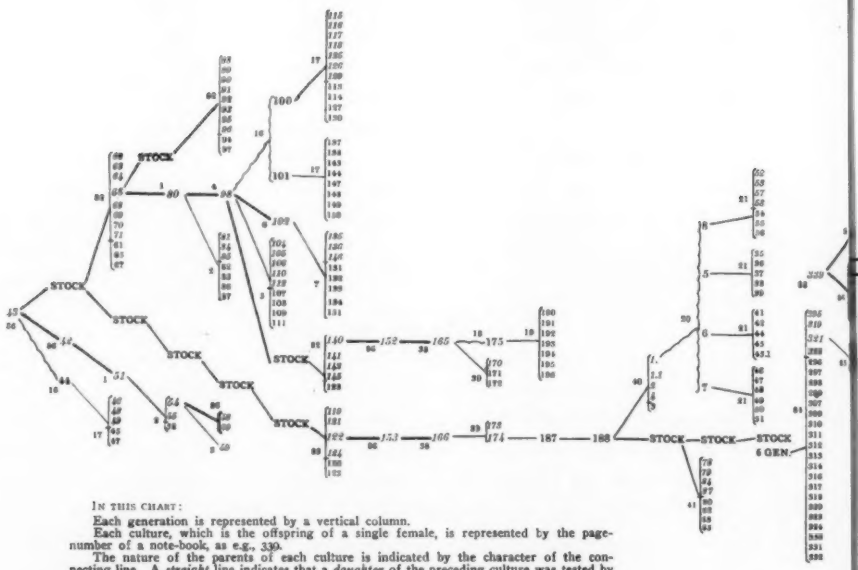
By breeding in each generation the exceptional daughters with the exceptional sons a line was maintained in which the entire set of sex-linked genes of the mother was handed down to the exceptional daughters and the entire set of the father to the exceptional sons.

That the XX eggs which developed into matroclinous exceptions had really been fertilized by normal sperm of the father was proved by the introduction into such daughters of autosomal genes from the father. The inheritance was uniparental with respect to the sex-linked genes, but biparental and wholly normal with respect to the autosomal genes. The fact that exceptional offspring inherit sex-linked characters from only one parent, but at the same time inherit the autosomal characters from both parents is explained if the sex chromosomes are the only chromosomes which have undergone non-disjunction, the ordinary chromosomes disjoining normally.

II

NEW DATA

All the facts presented in the first paper on non-disjunction can be explained, as has just been shown, on the assumption that the genes for sex-linked characters are carried by the X chromosomes and that these X chromosomes sometimes fail to disjoin, either primarily, or because of the presence of an extra Y (secondary non-disjunction). The new data now presented furnish evidence upon several other points suggested by



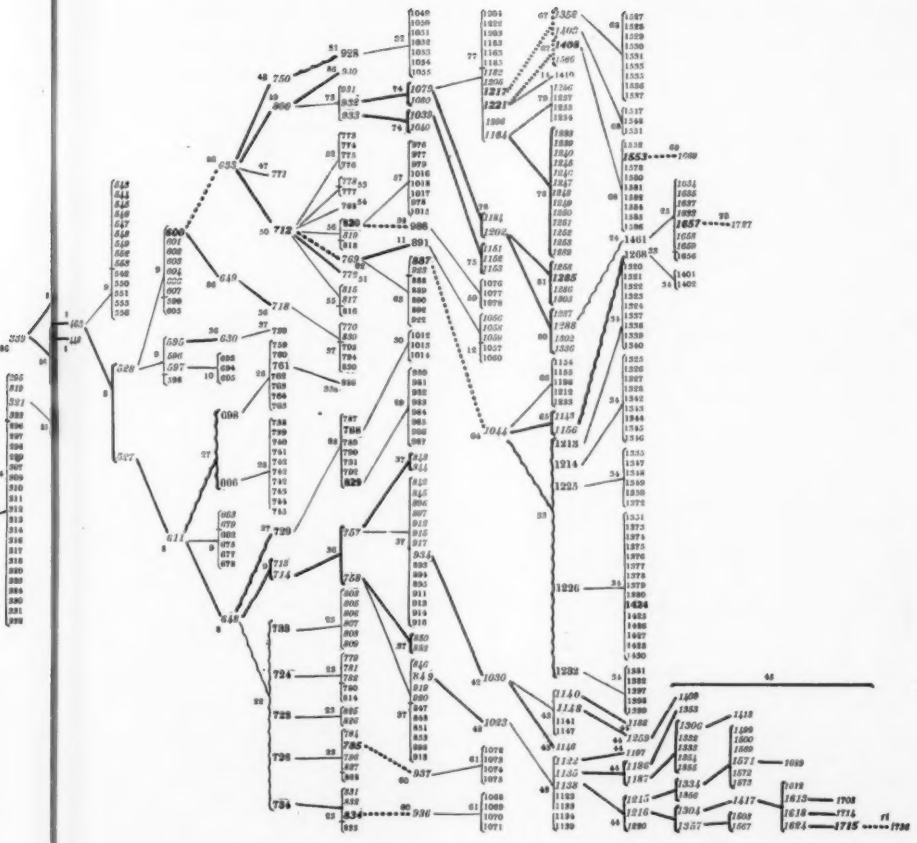
IN THIS CHART:

Each generation is represented by a vertical column.

Each culture, which is the offspring of a single female, is represented by the page-number of a note-book, as e.g., 339.

The nature of the parents of each culture is indicated by the character of the connecting line. A *straight* line indicates that a *daughter* of the preceding culture was tested by outcrossing to a male of another stock and gave the results of the culture to which the line leads. A *wavy* line indicates the outcrossing of a *son*. A *light* line indicates a *regular* daughter (————) or a *regular* son (~~~~~~~~). A *heavy* line indicates the outcrossing of an *exceptional* daughter (————) or of an exceptional son (————). A *heavy broken* line (- - - - -) indicates the outcrossing of an equational daughter. When more than one mating of a given kind was made the resulting cultures are included in a brace of the same character as the preceding line.

The nature of the offspring in each culture is indicated by the character of the type with which the culture-number is printed. The number prefixed to each brace gives the number of the table in this paper in which complete details of each culture are given. *Exceptions* appeared in those cultures printed in *italics*. *No exceptions* appeared in those cultures printed in Roman type. **Black face** type indicates the occurrence of *equational daughters*. Cultures from which matings were made are printed in LARGER TYPE.



the hypothesis, and also upon many points which the simple analysis failed to reveal.

The value of the evidence upon the main points has been increased by repeating a given test at different times and with different stocks or lines. Likewise the same test was often paralleled by starting more than one set of parents and then keeping separate the results from each. The tests have been further checked by using many different combinations of the most workable of the sex-linked characters. This method of distributing the tests over a great variety of material imparts to the conclusions a generality which they might not otherwise possess. One test in most of the sets is carried out much more extensively than the others in order to know fully the results given by some one individual of a given type.

It would be impossible to be brief and yet make entirely clear by text and table headings the relations between the parts of an experiment, when these parts are similar in nature but different in origin. Accordingly, reference may be made with advantage to the pedigree opposite page 14, which includes all the cultures considered in this paper.

THE PERCENTAGE OF SECONDARY EXCEPTIONS

In the first paper on non-disjunction (BRIDGES 1913 b) the results of about twenty-five cultures which gave secondary exceptions, gave the following totals:

TABLE I

A summary of the offspring of XXY females from BRIDGES *1913 b.*

Regular offspring		Exceptions		Total	Percent of exceptions
Daughters	Sons	Daughters	Sons		
2,929	2,677	133	158	5,877	4.6

The various secondary cultures reported in this paper bring these totals up to:

TABLE II

Total of secondary exceptions from BRIDGES *1913 b and this paper.*[*]

Regular offspring		Exceptions		Total	Percent of exceptions
Daughters	Sons	Daughters	Sons		
27,679	26,391	1,235	1,169	56,474	4.3

[*] These data are from tables 1-9, 11, 12, 17, 23, 25, 35a, 37, 38, 40, 41, 43-46, 48-51, 53, 55, 58, 60, 62-65, 67-71, 73-75, 78, 80-84, 86. The cultures known to be high non-disjunction are omitted.

In addition to these cultures there are several cultures in which the exceptional daughters could not be separated from the regular daughters, but in which the sons could be classified as regular and exceptional. The total of such cultures give:

TABLE III

Additional secondary cultures in which the sons only are separable.[7]

All daughters	Regular sons	Exceptional sons	Total sons	Percent of exceptions
1,681	1,496	77	1,573	4.3

The value given by these sons alone is the same as that given by the former cultures, namely, 4.3.

THE EQUALITY OF THE EXCEPTIONAL SONS AND DAUGHTERS OF XXY FEMALES

The total of the sons in tables II and III, is 29,133 of which 1,146 or 4.3 percent were exceptions. The daughters of table II total 28,914 of which 1,235 or 4.3 percent were exceptions. It is evident that the percent of exceptional daughters is the same as the percent of exceptional sons. This equality goes back to and proves an equality of XX and Y eggs (see figure 6, p. 13). These eggs are produced after XY synapsis when X and Y disjoin and the free X goes with the disjoined X, that is, by the XX-Y type of reduction. The XX eggs are produced when the Y goes to the polar body, and the Y eggs when both X's go to the polar body. Since it has been shown that there are as many XX as Y eggs it must be a matter of chance whether any particular egg receives the XX or the Y end of the spindle. In another section it will be shown that also in the other type of reduction, namely, the X-XY type, the free chromosome goes to the polar cell as often as it remains in the egg, so that the number of XY eggs is equal to the number of X eggs. Although there is no evidence that reductions are not taking place at random, there is proof that the synapses in an XXY female are strongly preferential, homosynapsis occurring much oftener than is expected on chance.

THE RELATION BETWEEN THE PERCENT OF HETEROSYNAPSIS AND THE PERCENT OF SECONDARY EXCEPTIONS

By reference to figure 6, page 13, it will be seen that the exceptional offspring are derived from germ-cells in which XY synapsis had

[7] These data are from tables 21, 36, 39, 42, 47, 79.

occurred. Not all the germ-cells in which heterosynapsis occurred lead to the production of exceptions, but there is a definite relation such that the percent of exceptions which should follow from a given percent of heterosynapsis can be calculated; and, vice versa, from the observed percent of exceptions the percent of cases of XY synapsis necessary to give rise to these exceptions can be determined. Let us assume that in an XXY female sixteen percent of all synapses are between Y and one or the other of the two X's. At reduction the Y and the X disjoin, and the free X goes to the same pole with X as often as with Y, so that in eight percent of cases the reduction is of the XX-Y type. After XX-Y reductions there are four percent of XX and four percent of Y eggs. Half of these eggs are fertilized by X and the other half by Y sperm, so that the zygotes are XXX, XXY, XY, and YY, each class comprising two percent of the whole number of zygotes. The XXX and YY zygotes die, while the XXY and XY zygotes give rise to exceptions, that is, $\frac{4\times 100}{96}$ or 4.2 should be the percent of exceptions surviving after sixteen percent of XY synapses. The percent of exceptions calculated from all the data is 4.3 which corresponds to 16.5 percent of heterosynapsis. This value was calculated by means of the formula $x = \frac{400\,y}{100 + y}$ in which x represents the percent of heterosynapsis and y represents the corresponding percent of exceptions. The converse formula $y = \frac{100\,x}{400 - x}$ is also useful. Curve *a* of figure 7 on page 18 gives for every percent of XY synapsis the corresponding percent of exceptions which should be observed. If all the synapses are between Y and one or the other X, then the percent of exceptions should be the upper limit of 33.3.

THE COEFFICIENT OF Y SYNAPSIS

If the synapses in an XXY female take place according to chance there should be twice as many XY as XX synapses. In order that the synapses have this distribution the three sex chromosomes must be equally potent in synapsis, that is, the synapsis coefficient of a Y in terms of an X must be 100 percent. With a synapsis coefficient of 100, 66.7 percent of the synapses should be between Y and an X, and there should result twenty percent of exceptions. Ordinarily not twenty but 4.3 percent of exceptions are produced, and the coefficient of Y must accordingly be far below 100. The relation between the coefficient of Y (c) and the percent of heterosynapsis (x) is expressed by the formulae $c = \frac{50\,x}{100 - x}$ and $x = \frac{100\,c}{50 + c}$.

In a previous section we found that the percent of heterosynapsis (x) was 16.5 so that the formula $c = \frac{50\,x}{100 - x}$ enables us to calculate the normal coefficient of Y as 9.9.

THE RELATION BETWEEN THE COEFFICIENT OF Y AND THE PERCENT OF EXCEPTIONS

Ordinarily the relation which is most useful is not that between the coefficient of Y and the percent of heterosynapsis but that between the coefficient of Y (c) and the percent of exceptions (y). By substituting the value for x just found, namely, $\frac{100\,c}{50 + c}$, for x in the formula $y = \frac{100\,x}{400 - x}$, y is found to be equal to $\frac{100\,c}{200 + 3\,c}$, and $c = \frac{200\,y}{100 - 3\,y}$. Curve *b* figure 7 gives graphically the relation between the coefficients of Y (up to 100) and the percents of exceptions.

A knowledge of the value of x, y, or c enables the other two values to be calculated from the two curves *a* and *b* or by the six formulae given.

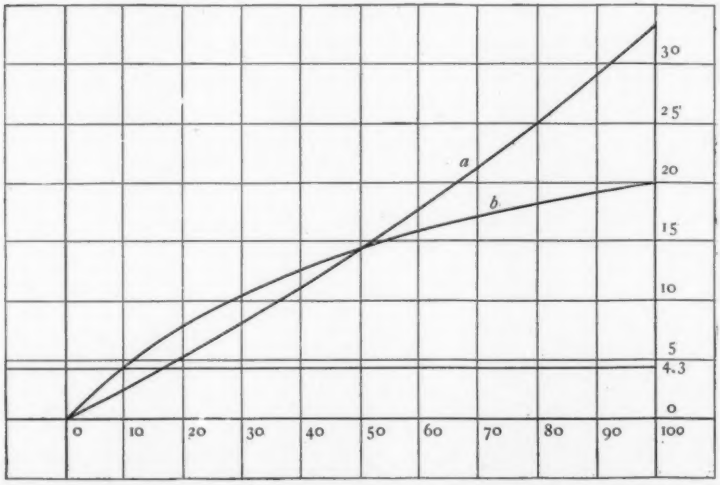


FIGURE 7.—Curve *a* shows the percentages of secondary exceptions (ordinates) corresponding to various percentages of XY synapsis (abscissas).

Curve *b* shows the percentages of exceptions (ordinates) corresponding to various coefficients of Y (abscissas). The line at 4.3 indicates the mean percent of observed secondary exceptions. This line intersects curve *a* at 16.5 and curve *b* at 9.9.

THE EQUALITY OF XXY AND XX FEMALES AMONG THE REGULAR DAUGHTERS OF XXY FEMALES

Figure 6 shows that the regular daughters, although all alike externally, must be supposed to be of two classes with respect to their sex chromosomes. One class (fig. 6, (3)) is XX and the other (4) is XXY. If the presence of the extra Y causes secondary exceptions to occur, then these XXY daughters should produce exceptions even though they themselves were not exceptions. If in reductions of the X-XY type, the X and the XY group remain in the eggs equally often, then these XX and XXY daughters should be in equal numbers, that is, half of the regular daughters should produce exceptions. That this is the case is shown by the experiments which follow.

The first regular daughters tested were those obtained by crossing two eosin exceptional females to wild males.

TABLE 1

The offspring given by two eosin exceptional females when outcrossed by two wild males.

No.	Regular Offspring		Exceptions		Percent of exceptions
	+♀	w^e♂	w^e♀	+♂	
51n	105	119	6	10	6.7
80n	83	93	6	4	5.4
Total	188	212	12	14	

Eosin is recessive, so that the regular daughters which are heterozygous for eosin do not differ in appearance from wild flies. These daughters must be tested by males which have some sex-linked character different from those already in the cross, otherwise we could not distinguish between the patroclinous exceptions and the regular sons. White bar males, which differ in two characters from any regular son, were used in testing these females.

The exceptional sons produced were white bar, and the exceptional daughters did not receive the dominant bar from the father and were accordingly wild type. The females tested proved to be of two sorts: five gave exceptions and five did not give exceptions. These females must have been of the types XXY and XX as anticipated.

One of the regular white-eosin bar daughters of culture 54n seemed to have a darker eye-color than her sisters. When she was bred to a

TABLE 2

The two kinds of results given by the regular daughters from table 1 when tested by white bar males.

No.	Regular daughters		Regular sons		Exceptions		Percent of exceptions
	w-w^e B′	B′	w^e	+	+♀	w B′♂	
54n	90	92	111	101	11	13	5.7
55n	17	26	17	21	—	1	1.2
81n	50	54	44	65	2	2	1.9
84n	31	34	27	33	1	1	1.6
85n	38	46	30	43	1	3	2.5
Total	226	252	229	263	15	20	3.5
52n	116	124	110	121	—	—	—
82n	40	43	28	32	—	—	—
83n	85	47	63	57	—	—	—
86n	54	57	53	45	—	—	—
87n	21	23	26	13	—	—	—
Total	316	294	280	268	—	—	—

wild male she gave only the expected eye-colors. This regular daughter proved to be XXY and the percentage of non-disjunction which she gave was unusually high.

TABLE 3

The offspring given by a regular w-w^{e}B′ daughter from culture 54n when tested by a wild male.

No.	Regular daughters		Regular sons				Exceptions		Percent of exceptions
			$\frac{w \quad B'}{w^e}$		$\frac{w \quad +}{w^e \quad B'}$				
	+	B′	wB′	w^e	w	w^{e}B′	w-w^{e}B′♀	+♂	
59n	28	34	26	18	13	13	11	12	14.8

The regular sons of table 3 show the amount of crossing over between white and bar. The original or non-crossover classes are white bar and eosin, which are written under the symbol $\frac{w \quad B'}{w^e}$. The crossover classes, white and eosin bar, are written to the right under the symbol $\frac{w \quad +}{w^e \quad B'}$.

In a second experiment an eosin female was outcrossed to white males.

The regular daughters, white-eosin compounds, were tested by mating to bar males.

TABLE 4

The offspring given by an eosin exceptional female when outcrossed to white males.

No.	Regular offspring		Exceptions		Percent of exceptions
	w-w°♀	w°♂	w°♀	w♂	
98n	56	65	6	7	9.7

TABLE 5

The two kinds of results given by the regular daughters of table 4 when tested by bar males.

No.	Regular offspring			Exceptions		Percent of exceptions
	B′ ♀	w♂	w°♂	w-w°♀	B′♂	
104n	60	22	19	3	5	7.3
105n	39	17	15	2	2	5.3
106n	56	24	31	5	3	6.7
110n	70	38	37	2	1	2.
112n	9	9	6	2	—	7.7
Total	234	110	108	14	11	5.2
107n	111	48	68	—	—	—
108n	95	53	53	—	—	—
109n	50	24	23	—	—	—
111n	103	56	53	—	—	—
Total	359	181	197	—	—	—

Of the females tested five produced exceptions, and four were normal (table 5). These nine females bring the total of tested females to twenty, of which eleven must have been XXY and nine XX in constitution.

The third experiment differed from the first only in that the wild type regular daughters were tested by bar males instead of by white bar males.

TABLE 6

The offspring given by an eosin exceptional female when outcrossed to wild males.

No.	Regular offspring		Exceptions		Percent of exceptions
	+♀	w°♂	w°♀	+♂	
102n	37	38	2	3	6.2

TABLE 7

The two kinds of results given by the regular daughters of table 6 when tested by bar males.

No.	Regular offspring			Exceptions		Percent of exceptions
	B'♀	w•♂	+♂	+♀	B'♂	
135n	71	32	44	2	1	2.
136n	59	23	20	—	2	2.
146n	103	39	48	3	1	2.1
Total	233	94	112	5	4	2.
131n	83	45	34	—	—	—
132n	49	22	29	—	—	—
133n	79	52	52	—	—	—
134n	105	44	54	—	—	—
151n	67	53	32	—	—	—
Total	383	216	201	—	—	—

Three daughters proved to be XXY, and five XX females, bringing the totals to fourteen of each kind.

The most extensive experiment of this type consisted of the tests by white bar males of the regular daughters from outcrosses of vermilion exceptional females to eosin males.

TABLE 8

The offspring given by vermilion exceptional females when outcrossed to eosin males.

No.	Regular offspring		Exceptions		Percent of exceptions
	+♀	v♂	v♀	w•♂	
401	87	95	2	1	1.6
448	53	65	2	1	2.5
463	48	37	2	—	2.3
527	111	95	2	—	1.
528	99	76	1	—	.5
611	136	113	3	2	2.
648	95	95	2	4	3.1
Total	629	576	14	8	1.5

TABLE 9

The two kinds of results given by the regular daughters of table 8 when tested by white bar males

No.	Regular daughters		Regular sons w° / v		w° + v		Exceptions +♀	Exceptions w B′♂	Percent of exceptions
	w-w° B′	B′	w°	v	w° v	+			
543	51	57	27	29	10	14	2	2	2.1
544	55	56	46	45	15	13	7	6	5.4
545	67	64	55	47	16	12	5	4	3.3
546	60	59	45	48	14	18	10	10	7.9
547	45	51	34	40	16	14	3	3	2.9
548	46	51	32	39	15	13	6	1	3.4
549	35	61	20	49	22	14	2	3	2.4
552	47	64	30	34	13	12	2	2	2.
553	59	70	38	20	17	20	5	2	3.
554	49	49	37	41	16	14	1	2	1.4
600	36	36	27	24	12	8	3	2	3.4 (w°♀ 1)
601	66	50	40	49	20	26	4	3	2.7
602	78	72	46	51	25	29	7	—	2.3
603	17	21	15	15	5	4	1	1	2.5
604	87	86	54	52	23	18	11	6	5.
606	77	82	62	53	28	21	2	4	1.8
607	58	78	45	49	17	18	1	1	.8
595	41	29	31	28	18	17	4	5	5.2
596	71	77	49	43	21	23	2	3	1.7
597	63	116	67	62	26	28	6	1	1.9
663	42	60	41	39	18	16	3	5	3.6
679	14	25	22	19	8	14	3	2	4.7
713	68	68	45	51	19	21	3	8	3.8
714	66	56	33	37	15	10	2	3	2.3
Total	1298	1438	741	964	409	397	95	79	3.2
542	67	49	45	34	26	14	—	—	—
550	38	37	41	28	15	14	—	—	—
551	63	59	51	56	20	19	—	—	—
555	66	73	45	47	19	18	—	—	—
556	66	59	35	38	11	10	—	—	—
599	27	26	28	16	12	5	—	—	—
605	82	92	50	69	26	30	—	—	—
598	29	24	15	17	8	8	—	—	—
662	73	56	46	57	17	19	—	—	—
675	77	97	51	38	30	25	—	—	—
677	25	30	23	18	6	7	—	—	—
678	21	22	15	12	4	11	—	—	—
Total	634	624	445	430	194	180	—	—	—

Of the thirty-six females tested in this experiment (table 9), twenty-four gave exceptional and twelve gave only regular offspring. This seemed a somewhat large departure from the expected equality, and for a time I suspected that there might be a real excess of XXY females; but that there is no such inequality of the two types is rendered certain by the experiments which follow. Culture 597 gave a peculiar ratio of 116 bar to 63 white-eosin bar daughters, where equality was expected. The cause of this ratio was not discovered, but the tests of three of the white-eosin bar daughters by cherry males showed that these females were XX (see table 10), and they may therefore be included in the final summary.

In another experiment an eosin vermilion exceptional female was outcrossed to wild males, with the result given in table 11. The regular wild type daughters were then tested by bar males. As shown in table 12, three of the daughters were XXŸ and two were XX females.

Many of the other experiments that were designed to test special points, have at the same time given information upon the ratio of XXY to XX daughters. In table 13, such data are collected under two headings, cultures giving exceptions and those free from exceptions. There were thirty-four XXY and forty-two XX daughters.

TABLE 10

The offspring given by three regular bar daughters from culture 597 when tested by cherry males.

	Regular daughters				Regular sons				Exceptional sons
	w	B′	w	B′	w	B′	w	B′	
No.	w-w^c	B′ +	w-w^c	B′	wB′	+	w	B′	w^c ♂
693	38	38	25	24	27	32	11	19	—
694	37	36	42	31	34	27	17	21	—
695	42	50	38	43	36	53	28	35	—
Total	117	124	105	98	97	112	56	75	—

TABLE 11

The offspring given by an eosin vermilion exceptional daughter when outcrossed to a wild male.

	Regular offspring		Exceptions		Percent of exceptions
No.	+♀	w^{e}v♂	w^{e}v♀	+♂	
891	34	34	—	—	—

TABLE 12

The two kinds of results given by the regular daughters of table 11 when tested by bar males.

	Regular daughters	Regular sons				Exceptions		Percent of exceptions
		w^e	v	w^e	v			
No.	B′	w^{e}v	+	w^e	v	+♀	B′♂	
1056	107	35	31	27	15	1	2	1.4
1058	52	27	17	10	10	—	4	3.3
1059	80	21	24	10	11	12	3	9.3
Total	239	83	72	47	36	13	9	4.4
1057′	134	30	26	18	20	—	—	—
1060	89	26	34	17	20	—	—	—
Total	223	56	60	35	40	—	—	—

TABLE 13

The two kinds of results given by the regular daughters from various other experiments when tested by appropriate males.

Giving exceptions (XXY)						No exceptions (XX)				
	Regular offspring		Exceptions			Regular offspring			Exceptions	
No.	♀♀	♂♂	♀♀	♂♂	Percent	No.	♀♀	♂♂	♀♀	♂♂
931	80	112	1	1	1.	730	80	73	—	—
932	77	74	11	13	13.7	777	156	159	—	—
933	113	112	11	8	7.8	818	141	154	—	—
778	144	166	1	1	.6	820	171	193	—	—
819	143	123	1	4	1.8	768	132	141	—	—
815	175	143	4	5	2.8	816	140	168	—	—
817	160	162	1	3	1.2	793	275	280	—	—
770	88	81	—	3	1.7	794	294	285	—	—
839	201	187	1	3	1.	830	141	184	—	—
887	102	92	1	2	1.5	888	105	98	—	—
923	67	65	1	—	.8	889	63	63	—	—
842	151	149	3	9	3.9	890	264	267	—	—
845	115	159	5	6	3.9	892	160	130	—	—
896	177	165	1	—	.3	922	136	141	—	—
897	163	154	2	3	1.6	893	108	111	—	—
912	178	186	4	5	2.4	894	110	123	—	—
915	185	161	—	1	.3	895	178	153	—	—
917	187	184	2	1	.8	911	210	190	—	—
934	69	99	4	1	2.9	913	182	188	—	—
846	117	97	4	1	2.3	914	106	106	—	—
849	129	146	11	12	7.7	916	190	225	—	—
919	191	185	3	6	2.3	847	173	172	—	—
920	82	90	4	—	2.3	848	185	171	—	—
1182	89	76	4	4	4.6	851	144	116	—	—
1205	26	20	3	4	13.2	853	166	152	—	—
1217	116	108	9	5	5.9	898	115	86	—	—
1221	97	79	7	6	6.9	918	67	46	—	—
1206	15	14	1	5	16.6	1203	91	70	—	—
1164	30	19	20	15	41.6	1183	78	76	—	—
1140	154	158	8	7	4.6	1163	108	108	—	—
1148	74	57	4	3	5.1	1185	85	110	—	—
1122	123	92	7	3	4.4	1154	40	49	—	—
1135	113	122	12	5	6.7	1155	190	149	—	—
1138	125	129	12	4	3.9	1198	139	108	—	—
						1212	78	62	—	—
						1141	93	102	—	—
						1147	143	122	—	—
						1123	49	86	—	—
						1133	74	72	—	—
						1134	104	79	—	—
						1139	43	47	—	—
						1233	196	201	—	—

Table 14 gives a few more cultures of the same nature as those given in table 13; but here the males used in testing were such that only among the sons could exceptions be distinguished from the regular offspring. Five females were XXY and six XX.

TABLE 14

The two kinds of results given by regular daughters;—cases where exceptions could be seen only among the sons.

Giving exceptions (XXY)					No exceptions (XX)			
	Regular offspring		Exceptions		Regular offspring			Exceptions
No.	♀	♂	♂	Percent	No.	♀	♂	♂
170n	36	23	1	4.2	171n	32	22	—
173n	112	78	4	4.9	172n	100	79	—
174n	49	57	4	6.1	1257	59	62	—
771	139	127	4	3.1	1255	91	73	—
1256	207	187	9	4.6	1254	83	59	—
					1410	71	77	—

As the summary in table 15 shows, there have been eighty cultures giving exceptions and seventy-nine free from exceptions.

TABLE 15

Summary of the XXY and XX cultures.

Table	XXY	XX
2	5	5
3	1	—
5	5	4
7	3	5
9	24	12
10	—	3
12	3	2
13	34	42
14	5	6
Total	80	79

If the daughters which give exceptions do so because of the presence of a Y chromosome, then it must be concluded from this evidence that the two types XXY and XX are equally frequent (see fig. 6). This means that in reductions of the X-XY type it is a matter of chance whether the extra Y remains in the egg or passes to the polar body, so that XY and X eggs are formed in equal numbers.

THE EQUALITY OF XYY AND XY MALES AMONG THE REGULAR SONS OF XXY FEMALES

In the last section, the daughters produced by the fertilization of X and XY eggs by X sperm were studied, and it was demonstrated that these eggs were equally numerous. A corresponding equality of regular sons of the two types XY (fig. 6 (7)) and XYY (8) must result from the fertilization of these eggs by Y sperm. It is far more difficult to determine this ratio of X to XY eggs by tests of the equality of XY and XYY sons than it is to test this same ratio by means of the XX and XXY daughters. Only enough of the regular sons have been tested to demonstrate the existence of the two types and to show that they are approximately in equal numbers.

SECONDARY NON-DISJUNCTION IN THE MALE

In the spermatogenesis of an XYY male there are two possible methods of synapsis, the XY and the YY types. If the synapses are not preferential but take place according to chance, there should be twice as many XY as YY synapses. At the reduction division the two chromosomes that have synapsed disjoin and go into different cells, while the unsynapsed chromosome goes equally often with each of the other two. Thus after XY synapsis, X and Y disjoin and the free Y goes as often with the Y (X-YY type) as with the X (XY-Y type). In spermatogenesis both cells produced by the reduction division give rise to sperm, so that the X-YY type must produce an equality of XY and Y sperm. As a result of reduction there are four classes of sperm, two small classes, X and YY, from heterosynapsis, and two classes, XY and Y, twice as large and coming from two sources. Obviously there is no chance for the production of exceptions in F_1 through the fertilization of the normal X eggs of an XX female by any of the four kinds of sperm of the XYY male. For example, as figure 8 shows, only the expected wild type daughters and sable sons result from the fertilization of an ordinary sable female by a vermilion XYY male.

The wild type daughters of an XYY male are expected to be of two kinds, XX daughters (figure 8, 1) from the X sperm, and XXY daughters (3) from the XY sperm. It has been shown that females having the constitution XXY produce secondary exceptions, both when they are matroclinous exceptions and when they are regular daughters. Of still another origin are these XXY daughters of an XYY male, but these also should produce secondary exceptions if the presence of the extra Y is the cause of such exceptions.

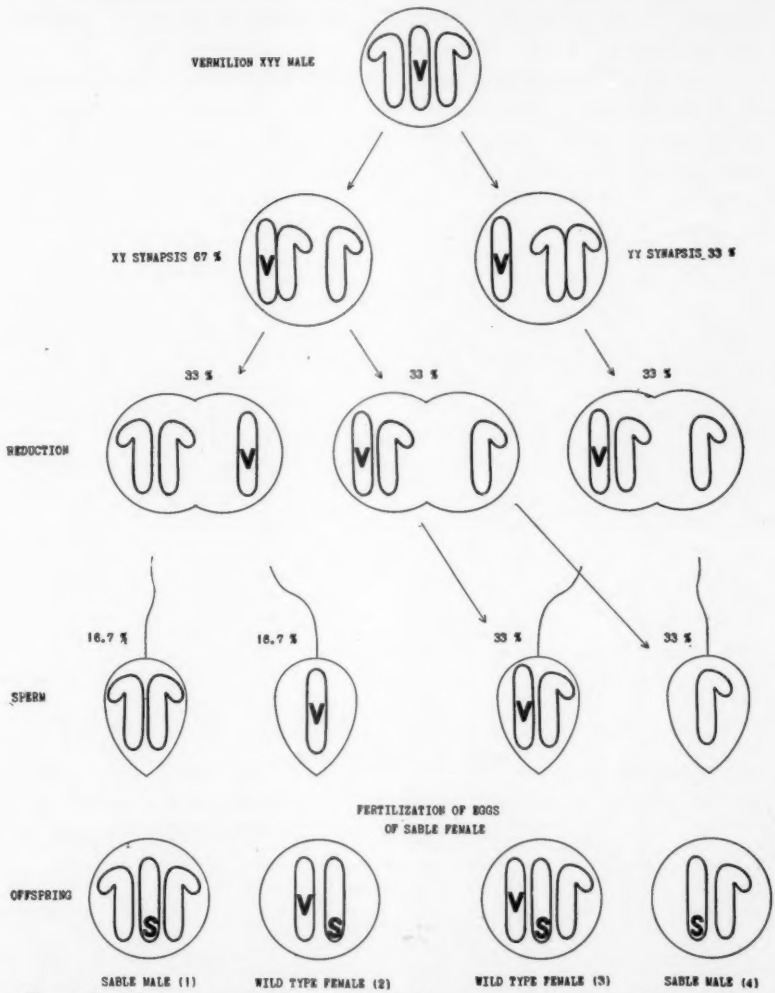


FIGURE 8. Diagram of secondary non-disjunction in the male. Four kinds of sperm are produced, but none of these lead to the production of exceptions in F_1.

If synapsis is according to chance, twice as many daughters of an XYY male should produce exceptions as do not. From the actual percentage of XX daughters a calculation of the amount of XY synapsis can be made.

In the first set of experiments to test the points just indicated, three eosin regular sons (from outcrosses of eosin exceptional females by white males) were outcrossed to white females. There were no exceptions in F_1, as is shown in table 16. Some of the white-eosin regular daughters from each culture were tested by white bar males. As table 17 shows, two of the three sets (*A* and *B*) gave exceptions, while the other set (*C*) gave no exceptions.

TABLE 16

The results given when three eosin regular sons were outcrossed to white females.

No.	w-w⁺♀	w⁺♂
44n	77	93
100n	157	122
101n	95	91
Total	329	306

TABLE 17

A. The two kinds of results given by the wild type daughters from culture 44n when tested by white bar males.

No.	Regular offspring				Exceptions		Percent
	wB′♀	w-w⁺B′♀	w♂	w⁺♂	w-w⁺♀	wB′♂	
46n	54	50	54	52	6	1	3.2
48n	54	48	57	54	5	3	3.6
49n	56	54	51	60	1	4	2.2
Total	164	152	162	166	12	8	—
45n	59	59	61	62	—	—	—
47n	68	72	58	55	—	—	—
Total	127	131	119	117	—	—	—

B. Daughters from culture 100n.

115n	44	32	36	33	9	4	8.2
116n	25	27	21	28	12	5	15.3
117n	52	55	66	51	7	16	9.4
118n	31	37	30	37	10	5	10.
125n	27	34	27	23	6	5	9.
126n	9	8	14	13	6	2	15.3
129n	39	35	45	26	9	3	7.6
Total	227	228	239	211	59	40	—
113n	36	53	51	57	—	—	—
114n	36	57	50	50	—	—	—
127n	40	43	45	38	—	—	—
130n	27	31	22	30	—	—	—
Total	139	184	168	175	—	—	—

C. Daughters from culture 101n

137n	43	40	39	42	—	—	—
138n	38	30	39	25	—	—	—
143n	37	31	30	30	—	—	—
144n	20	18	24	24	—	—	—
147n	43	38	24	43	—	—	—
148n	26	20	30	31	—	—	—
149n	37	43	22	30	—	—	—
150n	38	34	33	39	—	—	—
Total	282	254	241	264	—	—	—

In another experiment a single eosin regular son was outcrossed to bar females (table 18). Of the seven regular daughters tested by vermilion sable males, none gave exceptions (table 19).

TABLE 18

The offspring given by an eosin regular son when outcrossed to bar females.

No.	B'♀	B'♂
175n	67	75

TABLE 19

The results given by the wild type daughters from culture 175n when tested by vermilion sable males.

	Regular offspring		Exceptions
No.	♀♀	♂♂	v s ♂
190n	21	21	—
191n	56	50	—
192n	40	48	—
193n	67	66	—
194n	56	57	—
195n	59	48	—
196n	50	48	—
Total	349	338	—

In the next experiment four eosin regular sons were outcrossed to wild females (table 20).

Some of the wild type regular daughters from each of these cultures were tested by vermilion miniature males. As table 21 shows, one of the four sets gave exceptions, while the other three sets gave no exceptions.

TABLE 20

The offspring given by four eosin regular sons when outcrossed to wild females.

No.	+♀	+♂
8	94	95
5	74	64
6	70	69
7	76	68
Total	314	296

TABLE 21

A. The results given by wild type daughters from culture 8 when tested by vermilion miniature males.

	Regular offspring		Exceptions
No.	♀♀	♂♂	v m ♂
52	17	19	2
53	121	103	9
57	125	103	7
58	144	133	1
Total	407	359	19
54	126	109	—
55	60	44	—
56	90	86	—
Total	276	239	—
B. Daughters from culture 5.			
35	51	44	—
36	43	47	—
37	45	36	—
38	125	117	—
39	74	64	—
	338	308	—
C. Daughters from culture 6.			
41	44	52	—
42	23	28	—
44	57	49	—
45	46	43	—
45.1	73	75	—
Total	243	247	—
D. Daughters from culture 7.			
46	92	97	—
47	85	81	—
48	49	42	—
49	67	51	—
50	61	52	—
51	34	43	—
Total	388	366	—

In another experiment vermilion regular sons were outcrossed to eosin females (table 22), and the daughters were tested by bar males. All of the sets gave exceptions (table 23) showing that all five regular sons had been XYY.

TABLE 22

The results given by five vermilion regular sons when outcrossed to eosin females.

No.	+♀	w°♂
733	45	41
724	68	70
723	76	87
726	63	56
734	61	73
Total	313	327

TABLE 23

A. The results given by the regular daughters from culture 733 when tested by bar males.

No.	Regular daughters	Regular sons				Exceptions		Percent of exceptions
		w° v		w° + v				
	B′	w°v	+	w°	v	+♀	B′♂	
803	17	3	2	3	2	1	—	3.6
805	171	57	53	39	26	4	2	1.7
806	150	41	44	20	26	1	4	1.7
807	44	11	13	9	12	1	1	2.2
808	132	43	53	20	22	1	4	1.8
809	54	23	16	9	7	1	1	1.8
Total	568	178	181	100	95	9	12	
B. Daughters from culture 724 tested by white bar males.								
779	77 66	36	48	24	27	4	6	3.5
781	13 21	6	10	4	4	2	2	6.5
782	65 57	41	28	17	27	—	3	1.3
Total	155 144	83	86	45	58	6	11	—
780	59 60	27	27	14	13	—	—	—
814	37 46	28	27	21	35	—	—	—
Total	96 106	55	54	35	48	—	—	—

C. Daughters from culture 723 tested by white bar males.

825	66	38	33	29	14	17	4	—	2.
826	46	35	24	26	13	15	3	—	1.9
Total	112	73	57	55	27	32	7	—	

D. Daughters from culture 726 tested by white bar males.

784	65	67	37	40	18	18	3	3	2.4
785	97	84	69	77	32	36	6	7	3.2 (w*♀1)
786	110	113	68	61	40	34	4	4	1.8
827	72	92	55	56	29	25	4	4	2.4
Total	344	356	229	234	119	113	17	18	—
783	41	52	33	51	17	16	—	—	—
828	113	113	78	69	34	30	—	—	—
Total	157	165	111	120	51	46	—	—	

E. Daughters from culture 734 tested by white bar males.

831	36	34	17	27	17	11	—	1	.7
832	60	51	46	36	14	15	1	1	.9
834	11	8	9	6	5	4	1	—	2.3 (w*♀1)
Total	107	93	72	69	36	30	2	2	
833	13	20	11	10	4	3	—	—	—

Cultures 785 and 834 each produced an eosin female. These are examples of equational non-disjunction, and will be very fully discussed later.

In the final experiment, a yellow eosin vermilion regular son was outcrossed to wild females (table 24). The daughters were tested by bar males (table 25).

TABLE 24

The offspring given by a yellow eosin vermilion regular son when outcrossed to wild females.

No.	+♀	+♂
1461	69	73

TABLE 25

The offspring given by the wild type daughters from culture 1461 when tested by bar males

No.	Regular daughters	y w° v		y | w° v		y w° | v		y | v / w°		Exceptions		Percent of exceptions
	B′	y w°v	+	y	w°v	y w°	v	yv	w°	+♀	B′♂	
1634	55	21	36	—	—	6	17	—	—	3	6	6.2
1635	124	34	32	2	2	15	14	1	—	12	12	9.7
1637	135	29	46	—	1	12	22	—	1	13	14	9.9
1638	81	26	31	—	—	9	6	—	—	16	16	17.3
1657	161	38	48	1	1	21	13	—	—	12	15	9.0 (yw°♀1)
1658	99	41	43	—	—	11	13	—	—	17	23	16.2
1659	63	31	35	—	—	8	5	—	—	10	11	12.9
Total	718	220	271	3	4	82	90	1	1	83	97	
1656	36	14	16	—	—	4	4	—	—	—	—	—

All except one of the eight daughters tested gave exceptions. The percentages of exceptions given by the cultures of table 25 were unusually high.

TABLE 26

A summary of the tests of the regular sons classified as XYY if their daughters gave exceptions, and as XY if they gave none.

From XYY male			From XY male		
Origin	XXY daughters	XX daughters	Origin	XXY daughters	XX daughters
44n	3	2	101n	—	8
100n	7	4	175n	—	7
8	4	3	5	—	5
733	6	—	6	—	5
724	3	2	7	—	6
723	2	—			
726	4	2			
734	3	1			
1461	7	1			
9XYY♂♂	39	15	5XY♂♂	—	31

As shown in the summary of table 26, fourteen regular sons have been outcrossed and tested through tests of their daughters. Five of these sons must have been ordinary XY males, since none of their daughters gave exceptions. Nine sons must have been XYY in composition, since some of the daughters from each gave exceptions. This ratio of 9:5 must be considered as only a chance deviation from equality since these two classes of sons were produced by the same process which has been shown to produce two classes of daughters in equal numbers.

The results given by the daughters of the nine XYY males show that many more of these daughters must have been XXY than were XX. Out of a total of fifty-four such daughters tested, only fifteen, or twenty-eight percent gave no exceptions.

If the synapses in an XYY male take place according to chance, there should be 67 percent of XY synapsis (heterosynapsis) and 33 percent of YY synapsis (homosynapsis). Since in this section we have been testing only the daughters of XYY males, the calculation can be narrowed to the ratio of X to XY sperm which should follow various percents of XY synapsis. The X-bearing sperm from the cases of XY synapsis should be half XY and half X, while from homosynapsis all should be XY, that is, 33 percent of the X-bearing sperm should produce XX daughters and 67 percent XXY daughters. If all synapses are XY then 50 percent of the "female-producing" sperm are X. Among the female-producing sperm the percent of X sperm (y) which follow various percentages of heterosynapsis (x) can be found by the simple formula $y = \frac{x}{2}$. The realized value of 28 percent of X sperm suggests that the synapses are according to chance, which would give 33 percent of X sperm. In an XYY male the synapsis coefficient of X (c) in terms of Y (100) is expressed by the formula $c = \frac{50\,y}{50 - y}$.

THE CONSTITUTION OF THE PATROCLINOUS SONS OF XXY FEMALES

The usual method by which males are produced is by the fertilization of X eggs by Y sperm; but if our analysis of secondary non-disjunction is correct the patroclinous sons are produced by a method the exact reverse of this—*by the fertilization of Y eggs by X sperm* (see figure 6). Such males are XY in composition and should therefore behave in heredity exactly like ordinary males. They should be able neither to produce exceptions nor to transmit the power of producing exceptions.

In order to test this conclusion, three eosin exceptional sons (from

outcrosses of vermilion exceptional females to eosin males) were outcrossed to vermilion females. The offspring consisted of the expected wild type daughters and vermilion sons, as shown in table 27. Some of the wild type daughters from each male were tested by bar males, with the results shown in table 28.

TABLE 27

The offspring given by three eosin exceptional sons when outcrossed to vermilion females.

No.	+♀	v♂
698	116	94
666	96	91
729	105	82
Total	307	267

TABLE 28

A. The offspring given by wild type daughters from culture 698 when tested by white bar males.

	Regular daughters		Regular sons				Exceptions	
			w°	v	w°v	+		
No.	w-w°B′	B′					+♀	wB′♂
759	76	107	52	58	32	32	—	—
760	83	71	60	64	28	25	—	—
761	77	74	56	53	19	30	1	—
762	89	71	44	50	21	34	—	—
763	73	70	46	43	18	27	—	—
764	61	61	37	36	16	19	—	1 (Sterile)
765	81	75	61	67	36	18	—	1 (Sterile)
Total	540	519	356	371	170	185	1	2

B. Daughters from culture 666.

738	89	87	66	59	17	28	—	—
739	86	78	63	60	19	25	—	—
740	44	49	25	31	21	14	—	—
741	91	107	61	57	25	27	—	—
742	83	81	36	63	23	22	—	—
743	92	87	80	81	14	29	—	2
744	43	45	26	27	9	11	—	—
745	85	99	73	70	20	27	—	—
Total	613	633	430	448	148	183	—	2

C. Daughters from culture 729.

787	139	140	95	78	50	38	1	—	
788	124	114	83	73	42	30	1	—	
789	140	140	87	82	37	56	—	—	
790	121	133	98	74	51	27	—	—	
791	111	101	61	73	28	28	—	—	
792	113	108	87	80	40	45	—	—	
829	118	123	90	73	34	32	—	—	(w^e♀ 1)
Total	866	859	601	532	282	256	2	—	(w^e♀ 1)

Of the twenty-two daughters tested, fifteen produced no exceptions. In culture 743 there were two white bar males present on the first day of hatching; but unfortunately there is some doubt as to whether these males were parents which had escaped being thrown out, or were exceptional sons; the probability is that they were exceptions. There is no doubt that in six other cultures genuine exceptions occurred. The analysis had led to the inference that an exceptional son can not transmit to his daughters the power of producing exceptions. What then is the explanation of these real exceptions?

It seemed most probable that these exceptions were all primary, and not due to the presence of an extra Y in the females tested. It is very striking that in spite of the unusually large output of the cultures of table 28 only one (and this the somewhat doubtful one), produced more than a single exception. As already shown by the tables in which secondary non-disjunction occurs, such a very low production of exceptions would be most unusual for a like number of XXY females. In this respect these tables are in marked contrast to those in the last section.

The exception in culture 829 was an eosin daughter and is another example of the rather rare equational non-disjunction to be discussed later. In the case of culture 829 there is direct evidence showing that the mother did not have a Y chromosome and that the exception is primary. We have seen that half the daughters of an XXY female are themselves XXY and in turn produce exceptions. As shown in Table 29 the daughters from culture 829 did not fulfill this condition, and therefore the mother of culture 829 was not XXY.

Seven of the eight daughters of 829 gave no exceptions at all and must therefore have been simply XX in composition; the remaining daughter gave a single patroclinous son. This son proved to be absolutely sterile, from which test we may conclude that he was a primary exception from an XX mother; for, as will be shown later, primary

TABLE 29

The offspring given by the white-eosin bar regular daughters from culture 829 when tested by miniature males.

No.	Regular daughters	Regular sons: w/w° B′		Regular sons: w/w° | B′		Exceptions: w-w°B′♀	Exceptions: m♂
		wB′	w°	w	w°B′		
980	74	25	16	16	10	—	—
981	77	17	22	26	17	—	—
982	73	19	18	13	12	—	—
983	119	31	26	20	30	—	—
984	58	16	19	15	12	—	1 (Sterile)
Total	401	108	101	90	81	—	1
		All regular sons.					
985	156	156				—	—
986	119	115				—	—
987	113	102				—	—
Total	388	373				—	—

male exceptions with the formula XO are totally sterile. If this single exception was primary, then none of the eight daughters from culture 829 was an XXY female, and this result shows that the mother of 829 was simply XX; the chances are 255 to 1 that one of the eight daughters would have been XXY if the mother were XXY.

The same test that was used in the case of culture 829 was applied to culture 788, another of the seven cultures of table 28 which gave exceptions.

TABLE 30

The offspring given by the white-eosin bar regular daughters from culture 788 when tested by miniature males.

No.	Regular daughters	Regular sons: w/w° B′		Regular sons: w/w° | B′		Exceptions: w-w°B′♀	Exceptions: m♂
		wB′	w°	w	w°B′		
1012	151	35	27	35	27	—	—

All regular sons.

1013	26	28	—	—
1014	130	128	—	1 (Sterile)
Total	156	156	—	1

One of three daughters from culture 788 gave a single patroclinous son which was sterile and presumably therefore a primary exception (table 30). None of the three daughters were XXY and this suggests (chances 7 to 1) that their mother was not XXY.

The exceptional males of cultures 764 and 765, table 28, were both sterile and agree therefore with the expectation for primary exceptions.

These tests have shown that four out of the six undoubted exceptions were primary and lead to the belief that the other exceptions were also primary.

In a second experiment the exceptional son tested was a yellow male from an outcross of an eosin exceptional female to a yellow male. This son was outcrossed to a white female (table 31) and his wild type daughters were tested by bar males (table 32).

TABLE 31

The offspring given by an exceptional yellow son when outcrossed to white females.

No.	+♀	w♂
928	113	106

TABLE 32

The offspring given by the wild type daughters from culture 928 when tested by bar males.

	Regular offspring	Exceptions	
No.	♀♀ and ♂♂	+♀	B'♂
1049	20	—	—
1050	129	—	—
1051	237	—	—
1052	131	—	—
1053	39	—	—
1054	106	—	—
1055	58	—	—
Total	720	—	—

As shown by table 32 none of the seven daughters gave any exceptions.

In the final experiment, six sable forked exceptional sons were outcrossed to eosin vermilion females (table 33), and the daughters were tested by bar males. As shown by table 34, there is here no question of secondary non-disjunction. Only two cultures from a total of forty-six gave exceptions, and these were primary, as the results in the sister cultures prove.

TABLE 33

The offspring given by six sable forked exceptional sons when outcrossed to eosin vermilion females.

No.	+♀	w^e v♂
1213	61	43
1214	89	82
1225	63	46
1226	147	153
1232	97	80
Total	465	411

TABLE 34

A. The offspring given by the wild type daughters from culture 1213 when tested by bar males.

No.	Regular daughters	Regular sons																Exceptions	
		w^e v		w^e sf		w^e v sf		w^{e}v f		w^e		w^e s		w^{e}v s		w^e f			
		s f		v		+		s		v sf		v f		f		v s		+♀	B′♂
	B′	w^{e}v	sf	w^{e}sf	v	w^{e}vsf	+	w^{e}vf	s	w^e	vsf	w^{e}s	vf	w^{e}vs	f	w^{e}f	vs		
1320	100	21	27	12	14	4	6	–	–	1	3	–	–	–	–	–	–	–	–
1321	97	14	29	7	13	4	5	2	6	–	–	–	1	–	–	–	–	–	–
1322	44	14	9	3	6	1	2	6	3	–	1	–	–	–	–	–	–	–	–
1323	54	8	13	4	7	2	4	4	2	–	–	–	–	–	–	–	–	–	–
1324	116	27	27	2	–	3	3	7	5	1	–	2	–	–	1	–	–	–	–
1337	94	22	13	4	15	1	5	1	2	–	–	1	1	1	–	–	–	–	–
1338	34	1	14	1	5	–	–	–	3	–	–	–	1	–	–	–	–	–	–
1339	67	16	16	8	3	1	2	6	3	1	–	–	2	1	–	–	–	–	–
1340	104	18	26	9	16	3	6	5	–	–	2	–	–	–	–	–	–	–	–
Total	710	141	174	50	79	19	33	35	31	3	6	3	6	2	1	–	–	–	–

B. Daughters from culture 1214.

1325	113	38	20	–	3	3	4	4	7	1	–	–	3	–	–	-	-	1 -
1326	119	28	23	19	19	2	7	9	2	–	3	2	–	–	–	-	-	- -
1327	161	50	51	14	22	3	8	9	6	–	1	3	2	–	–	-	-	- -
1328	159	24	39	21	18	5	5	12	10	1	1	–	2	–	–	-	-	- -
1342	93	24	21	6	7	2	3	5	5	–	–	–	–	–	–	-	-	- -
1343	73	12	16	7	4	2	5	–	4	–	–	–	–	–	–	-	-	- -
1344	51	16	10	8	8	–	3	3	2	–	1	2	–	–	–	-	-	- -
1345	56	19	12	5	9	3	1	3	5	–	–	1	1	–	–	-	-	- -
1346	171	42	33	10	22	8	5	9	4	2	1	2	2	–	–	-	-	- -
Total	996	253	225	90	112	28	41	54	45	4	7	10	10	–	–	-	-	1 -

C. Daughters from culture 1225.

1335	72	37	24	6	15	4	3	5	3	–	–	–	1	–	–	-	-	- -
1347	131	32	20	15	23	2	8	5	3	–	–	1	2	–	1	-	-	- -
1348	184	34	49	23	20	4	7	8	2	2	1	1	1	–	–	-	-	- -
1349	172	39	30	13	20	10	7	3	7	–	–	2	–	–	–	-	-	- -
1350	228	46	49	19	27	2	9	12	7	1	2	–	1	–	1	-	-	- -
1372	194	43	31	9	19	2	12	10	9	1	–	1	–	–	1	-	-	- -
Total	981	231	193	85	124	24	46	43	34	4	3	5	5	–	3	-	-	- -

D. Daughters from culture 1226.

1351	183	37	48	16	19	8	12	10	4	1	2	2	2	–	–	-	-	- -
1373	117	29	15	6	6	–	4	3	4	–	1	–	–	–	–	-	-	- -
1374	103	24	20	6	10	–	5	8	3	–	1	–	1	–	–	-	-	- -
1375	89	19	12	2	12	1	6	4	2	–	–	–	1	–	–	-	-	- -
1376	110	37	17	3	18	2	3	3	7	1	–	1	–	–	–	-	-	- -
1377	123	33	26	15	24	5	7	5	7	2	2	3	4	1	-	-	-	- -
1378	71	14	5	6	6	3	6	5	–	–	–	2	1	-	-	-	-	- -
1379	101	35	17	14	17	8	6	9	6	–	1	1	–	-	-	-	-	- -
1380	178	44	46	25	27	6	6	13	12	5	–	1	5	-	-	-	-	- -
1424	59	17	23	2	7	2	4	4	4	–	–	2	1	-	-	-	1	- (w⁺v♀1)
1425	40	55	6	3	2	–	1	3	1	–	–	–	–	-	-	-	-	- -
1426	157	56	49	28	17	9	7	7	11	1	1	3	2	1	-	-	-	- -
1427	42	13	17	6	5	3	–	1	2	1	–	1	–	-	-	-	-	- -
1428	73	12	13	8	11	–	3	1	4	–	–	1	1	-	-	-	-	- -
1430	24	4	4	3	4	1	–	–	1	–	–	1	–	-	-	-	-	- -
Total	1470	379	319	143	185	48	70	76	68	11	8	18	18	2	-	-	1	- (w⁺v♀1)

E. Daughters from culture 1232.

1381	75	12	14	5	9	2	3	3	2	1	–	1	1	-	1	-	-	- -
1382	110	22	30	16	23	2	3	6	12	–	–	–	2	-	-	-	-	- -
1397	190	44	45	28	14	4	3	9	11	1	–	2	3	-	-	-	-	- -
1398	147	34	40	9	8	3	3	3	10	1	–	1	1	-	-	-	-	- -
1399	198	65	53	37	26	6	8	13	10	11	–	4	2	-	-	-	-	- -
Total	720	177	182	95	80	17	20	34	45	4	–	8	9	-	1	-	-	- -

F. Daughters from culture 1268.

1401	114	22	22	12	17	5	10	6	3	1	1	4	3	-	1	-	-	- -
1402	142	39	32	10	11	3	6	4	4	3	–	–	1	1	-	-	-	- -
Total	256	61	54	22	28	8	16	10	7	4	1	4	4	1	1	-	-	- -

TABLE 35

Summary of the results given by the daughters of exceptional sons.

Parentage	Exceptions absent	Primary exceptions
698	4	3
666	7	1
729	4	3
728	7	—
1213	9	—
1214	8	1
1225	6	—
1226	14	1
1232	5	—
1268	2	—
Ten tested	66	9

As the summary of table 35 shows, no exceptions at all appeared in sixty-six of the seventy-five cultures from daughters of exceptional sons. The exceptions which appeared in the other nine cultures may in one case be due to error (though this is not probable), and in the case of six others tests have been made which show the exceptions to have been primary. If all these exceptions were primary, as there seems no reason to doubt, then we may conclude that the exceptional sons do not transmit the power of producing secondary exceptions, and that their composition is simply XY.

XO MALES AND PRIMARY NON-DISJUNCTION

It is evident from the results of the previous section that primary non-disjunction occurring in the female is not extremely rare. There were twelve primary exceptions among the 20,484 flies which were descended from patroclinous sons. This is one in 1700. The actual number of occurrences of primary non-disjunction must have been twice as frequent since half the non-disjunctional eggs are lost as XXX and YY zygotes.

All exceptional daughters produced by primary non-disjunction should receive XX from the mother and Y from the father and should therefore produce secondary exceptions. The primary matroclinous daughter in culture 761, table 28, was tested upon this point by outcrossing to white bar males. The offspring showed that she was XXY as expected.

TABLE 35A

The offspring given by a primary matroclinous wild type daughter from culture 761 when tested by white bar males.

No.	Regular daughters		Regular sons				Exceptions		Percent of exceptions
			w^e / v		w^e + v				
	w-w^eB′	B′	w^e	v	w^ev	+	+♀	wB′♂	
886	88	79	83	47	29	38	5	7	3.2

The fact that XO males are totally sterile is of unusual interest since it is the first indication that the Y chromosome is something more than a gear wheel in the mechanism of synapsis and reduction. The evidence of this paper proves that the Y has no effect upon the sex or the sex-linked characters of either the male or female, but that the Y does play some positive rôle is proved by the fact that XY males are fertile and XO males are sterile.

THE INDEPENDENCE OF NON-DISJUNCTION

So far, the evidence has been treated upon the basis that secondary non-disjunction is caused by the presence of an extra Y chromosome, but practically all of the data given thus far might equally well be explained by the assumption of a *dominant* sex-linked gene. This hypothesis could be easily tested by finding the linkage relations of such a gene with other sex-linked genes. The amounts of crossing over between non-disjunction and each other gene should be consistent with its occupying a definite position in the X chromosome. The actual experiment showed that non-disjunction is independent of all the known sex-linked genes and can not therefore be due to a gene carried by the X chromosome.

On this view, a vermilion exceptional female must be supposed to have the gene for non-disjunction in at least one of her two X chromosomes. If such a female is mated to an ordinary eosin male, all the regular daughters should be heterozygous for eosin and vermilion and half should be heterozygous for non-disjunction also. The daughters carrying non-disjunction can be picked out because they give exceptions. Table 9 already given on page 23 gives the tests of many such females.

Any exceptional daughter given by such a culture should be an exact duplicate of her mother, that is, she should be heterozygous for eosin, vermilion, and non-disjunction. The offspring of such a female will

show by the proportions of the various classes the amount of crossing over between non-disjunction and eosin and between non-disjunction and vermilion. We can easily enough classify the offspring with respect to the eye-colors eosin and vermilion, but to determine whether any particular fly carries non-disjunction it is necessary to test for exceptions.

The first step of the experiment to test the linkage of non-disjunction was to cross an exceptional female from table 9 by an eosin tan vermilion male.

TABLE 36

The offspring given by four wild type exceptional daughters heterozygous for eosin, vermilion, and non-disjunction when crossed to eosin tan vermilion males.

	Daughters				Regular sons				Exceptional sons	
	w⁺ / v		w⁺ | v		w⁺ / v		w⁺ | v			
No.	w⁺	v	w⁺v	+	w⁺	v	w⁺v	+	w⁺ t v	Percent
630	29	29	19	20	21	19	8	6	11	16.9
718	27	25	19	20	13	18	7	9	1	2.1
757	65	59	29	45	76	68	23	29	8	3.9
758	61	64	37	35	53	62	34	37	7	3.6
	182	177	93	111	163	167	72	81	27	5.3

The offspring of four such matings are given in table 36. The wild type daughters in this table are of two kinds, namely, exceptional daughters which like the mother are heterozygous for eosin and vermilion, but which do not carry tan (since they received no X from the father), and regular daughters which do carry tan. This last class results from crossing over between eosin and vermilion and each fly must be tested to see whether it carries non-disjunction or not. Table 37 gives the results of the tests of thirty-five of the wild type daughters from table 36.

TABLE 37

The three kinds of results given by the wild type daughters from table 36 when tested by white bar males.

(A) Daughters giving exceptions.

	Regular daughters		Regular sons								Exceptions		
			w° t v		w° | t v		w° t | v		w° | t | v				
No.	w-w°B′	B′	w°tv	+	w°	tv	w°t	v	w°v	t	+♀	B′♂	Percent
770	39	49	23	26	15	10	2	4	1	—	—	3	1.7
839	111	91	52	57	30	42	2	4	—	—	1	3	1.
842	80	71	54	56	12	15	9	2	1	—	3	9	3.9
845	57	58	48	61	24	16	2	8	—	—	5	6	3.9
846	57	60	38	41	8	6	3	1	—	—	4	1	2.3
849	69	60	52	54	20	12	1	6	1	—	11	12	7.7
896	89	88	67	50	17	19	11	1	—	—	1	—	.3
897	85	78	54	56	13	21	6	4	—	—	2	3	1.6
912	85	93	68	68	27	19	4	—	—	—	4	5	2.4
915	100	85	62	57	17	15	5	5	—	—	—	1	.3
917	97	90	60	76	23	19	5	1	—	—	2	1	.8
919	106	85	59	67	20	26	5	8	—	—	3	6	2.3
920	48	34	31	28	9	15	3	3	—	1	4	—	3.4
934	32	37	24	47	9	12	3	4	—	—	4	1	2.9
Total	1055	978	692	744	244	247	61	51	3	1	44	51	2.3

(B) Daughters giving no exceptions.

730	40	40	25	37	—	5	5	1	—	—	—	—	—
793	129	146	99	94	29	34	16	7	—	1	—	—	—
794	147	147	90	83	49	46	10	6	—	1	—	—	—
830	73	68	57	57	32	33	3	2	—	—	—	—	—
847	93	80	64	63	19	21	—	5	—	—	—	—	—
848	100	85	63	59	22	15	6	5	1	—	—	—	—
851	64	80	51	37	19	7	1	1	—	—	—	—	—
853	87	79	53	55	18	12	7	6	1	—	—	—	—
893	60	48	42	35	8	12	6	6	1	—	—	—	—
894	58	52	53	39	13	14	1	2	1	—	—	—	—
895	86	92	66	50	18	14	4	7	—	—	—	—	—
898	57	58	37	28	11	9	1	—	—	—	—	—	—
911	116	94	59	79	20	23	5	4	—	—	—	—	—
913	85	97	66	72	18	18	8	6	—	—	—	—	—
914	45	61	39	50	4	7	3	3	—	—	—	—	—
916	91	99	77	77	24	39	4	4	—	—	—	—	—
918	39	28	20	10	11	4	—	—	1	—	—	—	—
	1370	1354	962	925	315	312	80	59	5	2	—	—	—

(C) Wild type daughters which were exceptions.

No.	Regular daughters		Regular sons				Exceptions		
			$\frac{w^e}{\quad\quad v}$		$\frac{w^e \quad\quad v}{+}$				
	w-w^{e}B′	B′	w^e	v	w^{e}v	+	+♀	wB′♂	Percent
843	98	83	61	71	23	12	9	8	4.7
844	62	75	50	48	16	21	9	12	7.2
850	64	60	43	40	13	24	3	6	3.6
852	29	31	17	33	6	6	2	6	6.2
Total	253	249	171	192	58	63	23	32	5.3

Of the thirty-five daughters whose tests appear in table 37, four (37 *C*) were matroclinous exceptions and useless for our present purpose. Of the thirty-one females which were crossovers between eosin and vermilion, fourteen, that is, about half, gave exceptions (37 *A*). Non-disjunction entered this experiment in company with vermilion, so that in the production of each of these fourteen females, which did not have vermilion (i.e., were wild type), but which did not produce exceptions there must have been a crossover between vermilion and non-disjunction. There are three general regions in which the gene for non-disjunction might be assumed to be located, namely, to the left of eosin, between eosin and vermilion, and to the right of vermilion. If we assume that non-disjunction is to the right of vermilion then the genes entered the cross in the following way $\frac{w^e}{\quad}$ $\frac{\quad}{v}$ and a wild type female which produced exceptions must be a double crossover ($\frac{w^e \quad + \quad v \quad +}{N}$, the egg represented by the lower symbol). A glance at any of the tables, for example tables 45A and 45D, in which double crossing over is recorded will demonstrate how rare an occurrence double crossing over is. But in this experiment we should have to conclude that the double crossovers (14) are practically as numerous as the single crossovers between eosin and vermilion (17), a result that is impossible on the assumption that non-disjunction is due to a gene whose inheritance is similar to the inheritance of other sex-linked genes. We must therefore conclude that a gene for non-disjunction does not lie in the portion of X to the right of vermilion. Likewise if non-disjunction lies to the left of eosin the seventeen wild type females which did not give exceptions must be double crossovers, a frequency of double crossing

over so great as to preclude the possibility of non-disjunction lying to the left of eosin. But if non-disjunction lies about midway between eosin and vermilion then neither of the two kinds of wild type females should be a double crossover. The fourteen normal females should represent single crossing over between eosin and non-disjunction ($\frac{w^e}{\quad} + \frac{N'}{\quad} {}^v$) and the seventeen non-disjunctional females should represent single crossing over between non-disjunction and vermilion ($\frac{w^e}{N'} + \frac{\quad}{\quad} {}^v$). This evidence, then, does not exclude the assumption that a gene for non-disjunction lies about midway between eosin and vermilion, though it does exclude the possibility of such a gene lying in any other position in the X. The evidence in the next section will show that non-disjunction can not lie even in this position midway between eosin and vermilion.

On the other hand the evidence in this section is entirely consistent with the assumption that the cause of the secondary exceptions is the presence of the Y chromosome in an XXY female. Exactly this realized result is expected if non-disjunction is independent of all sex-linked genes. The equality of wild type daughters carrying non-disjunction and free from non-disjunction is due to the free assortment of the Y from that X with which it entered the XXY zygote, and proves the important point that the Y synapses with and assorts from the two X chromosomes (X and X′) according to chance.

ATTEMPTS TO OBTAIN PURE STOCK OF NON-DISJUNCTION

The history of the various attempts to obtain a stock where every female should give exceptions when outcrossed, furnishes a new line of evidence to prove that a sex-linked gene cannot be the cause of secondary non-disjunction.

The first view that had presented itself was that non-disjunction was due to a recessive gene carried by the X chromosome. Since both the X chromosomes of a non-disjunctional female are transmitted intact to her exceptional daughters, every exceptional daughter would remain homozygous for such a gene. Likewise all the regular sons of such females should receive the gene. If this were true, mating an eosin exceptional female to her eosin regular brothers would give the required stock. However, tests of females of a stock obtained in this way showed that only about three quarters of them produced exceptions.

A possible explanation of this failure to obtain pure stock came with the discovery that half of the regular daughters of an exceptional female produce exceptions. The cause of the phenomenon was then thought to be a dominant rather than a recessive gene.

In attempting to secure a pure stock, a general method was used which enables one to follow a given chromosome through some generations with the certainty that it will emerge intact—that whatever genes were in it originally will still be in it, and that no others will have been switched in by crossing over.

The whereabouts in the X chromosome of the gene for non-disjunction was assumed to be unknown, so that the method was devised to guard the whole length of the X chromosome against escape of the gene while the stock was being made. The first step was to mate an eosin exceptional female, by hypothesis heterozygous for non-disjunction, to a yellow vermilion bar male:

TABLE 38

The results given by two eosin exceptional daughters outcrossed to yellow vermilion bar males.

No.	Regular offspring		Exceptions		Percent of exceptions
	B'♀	w°♂	w°♀	yvB'♂	
165n	68	75	7	4	7.1
166n	66	56	11	11	15.3
Total	134	131	18	15	

Half the regular bar daughters of this cross have a maternal chromosome bearing eosin and non-disjunction and can be picked out because they give exceptions. In table 39 are the results given by five

TABLE 39

The two kinds of results given by five bar regular daughters from table 38 when tested by yellow vermilion forked males.

No.	Daughters	Regular sons	Exceptional sons	
			yvp♂	Percent
170n	36	23	1	4.2
173n	112	78	4	4.9
174n	49	57	4	6.6
Total	197	158	9	—
171n	32	22	—	—
172n	100	79	—	—
	132	101	—	—

daughters when tested by yellow vermilion *forked* males (forked is only half a unit from bar). Eosin sons which have retained non-disjunction can be selected from the cultures which gave exceptions by the fact that they show neither yellow nor vermilion nor bar. Any eosin son which shows one or more of these characters does so because his X chromosome has undergone crossing over, whereby a section (which may be that section carrying non-disjunction) has been replaced by a foreign section carrying these other genes but not non-disjunction.

The characters yellow, vermilion, and bar had been chosen because they would reveal crossing over at practically every point along the chromosomes. Yellow is at the zero end of the chromosome while bar is quite close to the other end. Vermilion is at the middle and would reveal the comparatively rare double crossing over. When double crossing over occurs, a section from the middle of the guarded chromosome would be replaced by a corresponding section from the middle of the other or guarding chromosome.

The daughters which show neither yellow nor vermilion nor bar will likewise have the eosin non-disjunction chromosome intact. The next step was then to mate such a daughter by the eosin male which had been selected. From their offspring the eosin sons which have non-disjunction can be again selected by the fact that they show neither yellow nor vermilion nor forked. The eosin daughters received non-disjunction from the father and also from the mother in all those cases where crossing over has not taken place. Therefore any eosin daughter which gives no yellow or vermilion or forked sons when mated to the selected eosin brother would give a stock homozygous for eosin and non-disjunction. From among several pairs of eosin daughters by eosin brothers one pair which gave only eosin offspring was chosen to supply the required stock.

To test the purity of the stock obtained in this way some of the eosin females were mated to wild males. Four of the five females gave exceptions but one did not (table 40).

TABLE 40

The offspring given by five eosin females of a special stock of non-disjunction when tested by wild males.

No.	Regular offspring		Exceptions		Percent of exceptions
	+♀	w°♂	w°♀	+♂	
1	62	50	—	4	3.5
1.1	73	64	—	2	1.4
2	19	10	—	3	9.4
4	62	36	4	3	6.6
Total	216	160	4	12	
3	73	62	—	—	—

From the next generation of the same stock, eosin females were again tested by wild males with the result that four of the nine females failed to give exceptions (table 41).

TABLE 41

The offspring given by nine females from the second generation of a special stock of non-disjunction when tested by wild males.

No.	Regular offspring		Exceptions		Percent of exceptions
	+♀	w°♂	w°♀	+♂	
78	65	78	—	1	.7
79	100	73	1	1	1.1
84	50	61	3	5	6.7
86	35	31	2	—	2.9
87	82	32	1	—	.9
Total	332	275	7	7	
80	78	90	—	—	—
82	111	54	—	—	—
83	52	60	—	—	—
85	88	64	—	—	—
Total	329	268	—	—	—

It is evident that the stock was not pure for non-disjunction. The method used was rigorous enough to prove that the failure to produce a stock homozygous for non-disjunction is due to the fact that there is no such thing in the X chromosome as a gene for secondary non-disjunction. The gap in the evidence left open in the last section is completely closed by this method, for the point midway between eosin and vermilion is guarded equally with the whole of the X chromosome.

(To be continued)

THE NUMERICAL RESULTS OF DIVERSE SYSTEMS OF BREEDING[1]

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INTRODUCTION

To work out without formulae the results of almost any system of breeding long continued is a most laborious and time-consuming task, while a formula serves as a machine; one puts in the data and grinds out the results without labor. The present paper is a collection of formulae for the results of most of the usual systems of breeding. The systems dealt with are: random mating, assortative mating of domi-

[1] From the Zoölogical Laboratory of the Johns Hopkins University.

nants with dominants, recessives with recessives; selection of dominants alone; self-fertilization; and the various systems of inbreeding (of which a considerable variety exists). The present formulae give the distribution of single characters, sex-linked or not sex-linked, in continued breeding by such systems. A later paper will deal similarly with the cases where there are two or more characters, independent or linked together in various degrees. Most of the formulae are here presented for the first time; the few previously worked out are included in order to make as complete and systematic a collection as possible.

Strict Mendelian heredity presents a rigid scheme; its results can be computed; it is a problem in arithmetic. To work out its numerical formulae does not involve taking ground on the question whether all inheritance actually is of this rigid character. To discover whether the results of inheritance are continually subject to evolutionary change, as some hold, it is necessary to know first what would be the results without such evolutionary change. The results of actual experimentation may then be compared with what the formulae demand, in order to determine whether the processes of nature are or are not equally rigid.

The type of the formulae here set forth is the well-known 1 : 2 : 1 ratio observed in the second filial generation when two individuals having such factors as *AA* and *aa* are crossed. Recent developments in the study of heredity have introduced some complications into the relations; sex-linked characters follow rules of their own, and linkage greatly modifies the rules of distribution before known. These taken with the various systems of breeding give results requiring a variety of formulae for their working out.

The formulae for the first and second filial generations after making a cross between two parents differing in one or a few factors are of course well known. PEARSON (1904) shows that the formula $\frac{1}{4}\ AA + \frac{1}{2}\ Aa + \frac{1}{4}\ aa$ holds for any generation for the progeny by random mating of *AA* and *aa*, and HARDY (1908) has made the same point. The present writer (JENNINGS 1912) gave a formula for the results of self-fertilization. PEARL (1914 a) worked out the results of inbreeding of brother and sister among the progeny of $AA \times aa$, and gave a formula (PEARL 1914 b) showing how the constitution of any generation can be derived when that of the preceding generation is known. I gave a general formula for the proportion of homozygotes in any generation in this case (JENNINGS 1914), a formula superseded by the simpler ones in the present paper. FISH (1914) has studied the results of inbreeding, both of parent by offspring and of brother by sister; he gives the per-

TABLE 1. *Fundamental series in Mendelian breeding.*

	How formed																					
n	x + 1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Formula
B	2x	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288	1048576	2^n
C	2x + 1	0	1	3	7	15	31	63	127	255	511	1023	2047	4095	8191	16383	32767	65535	131071	262143	524287	1048575	$2^n - 1$
D	2x − 1		2	3	5	9	17	33	65	129	257	513	1025	2049	4097	8193	16385	32769	65537	131073	262145	524289	$2^{n-1} + 1$
E	2x + 1			2	5	11	23	47	95	191	383	767	1535	3071	6143	12287	24575	49151	98303	196607	393215	786431	$2^{n-1} + 2^{n-2} - 1$
F	Sum of two preceding	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181	6765	
G	2x + 1, then 2x − 1	0	1	1	3	5	11	21	43	85	171	341	683	1365	2731	5461	10923	21845	43691	87381	174763	349525	$(= B_{n-1} - G_{n-1})$
H	G − F	0	0	0	1	2	6	13	30	64	137	286	594	1221	2498	5084	10313	20858	42094	84797	170582	342760	
I	B − G − F	1	0	2	3	8	16	35	72	150	307	628	1276	2587	5228	10546	21235	42704	85784	172179	345344	692286	$(= G_{n-1} - F_n)$
J	$B_n - F_{n+1}$	0	1	2	5	11	24	51	107	222	457	935	1904	3863	7715	15774	31781	63939	128468	257963	517523	1037630	
K	$B_n - F_{n+2}$	0	0	1	3	8	19	43	94	201	423	880	1815	3619	7582	15397	31171	62952	126891	255379	513342	1030865	$(= J_n - F_n)$
L	$B_n - F_{n+1} - G_{n-1}$		2	2	6	11	24	48	99	200	406	819	1652	3324	6683	13420	26930	54003	108240	216856	434323	869632	
M	$3B_n - F_{n+2}$	2	4	9	19	40	83	171	350	713	1447	2928	5911	11911	23966	48165	96707	194024	389035	779667	1561918	3128017	$(= B_{n+1} - K_n)$

centages of homozygotes in the latter case up to 25 inbreedings.[2] Other work of this sort is not known to me.

The questions which the formulae are designed to answer may be illustrated as follows: Suppose that one parent has the factors *AA*, the other *aa;* and that after their crossing there is assortative mating (dominants with dominants, recessives with recessives) for seven generations. What will then be the constitution of the population; what proportion will be *AA*, what *aa*, what *Aa*? Further, if such mating is indefinitely continued, what are the limits to which the proportions of each of the three classes may finally attain? And how many generations must such breeding be continued before the limiting proportions are, for practical purposes, attained? Suppose that the parents differ, not in one pair of factors alone, but in several, how are these questions to be answered for each pair separately, and for the various possible combinations of them?

It should be emphasized of course that the results set forth in the formulae here given, hold with some degree of precision only if the experiment is carried out on an extensive scale, with many lines of descent. When the breeding is carried out with few lines of descent, the results given by the formulae are merely the most probable results.

We shall deal in part I with a single pair of typical Mendelian factors; in part II with single sex-linked factors.

We shall represent any single pair of alternative factors by *A* and *a*, in the three possible combinations *AA*, *Aa* and *aa*.

It is clear that the outcome of any system of breeding depends not only on the system followed, but also on the constitution of the parents at the beginning. For many (not all) types of breeding the beginning is a cross between two parents. The matings of two parents may be *AA* × *AA; aa* × *aa; AA* × *aa; AA* × *Aa; Aa* × *Aa*, or *A*a × *aa*. The first two matings require no consideration, since by any system of breeding all the progeny are like the parents. The most important differing parental types are *AA* × *aa* and *AA* × *Aa;* these give diverse results, while from other crosses the results are similar to one or the other of these. We shall specify in each case both the system of breeding followed and the constitution of the parents.

When breeding is continued according to a given system for many generations, several types of results may be distinguished, so far as concerns the constitution of the population, and the formulae expressing it:

[2] FISH states, by what is evidently a slip, that the percentages are of heterozygotism.

(a) In some cases the proportions (and formula) remain the same for all generations. It is mainly in random mating that this is the case. Examples are found in sections (1), (4), (37), etc.[3]

(b) The constitution of the population changes from generation to generation. In such cases it is often possible to give a simple formula based upon n, the number of generations; from it the result for any generation can be directly computed. Examples are (13), (16), (17), etc.

(c) In other cases the proportions of the population give a series of values, not easily expressible as simple functions of n alone. These cases are the most numerous and most interesting; they will be dealt with further presently.

(d) In some cases a general formula for any generation taken by itself is not obtained; the best that we can do is to give a formula for determining the constitution of any generation when that of the immediately preceding generation is known. Only a few cases of this sort have been found among those here dealt with, for example, (8), (53), (54), (55), etc.

FUNDAMENTAL SERIES

As just mentioned ((c), above), in most cases the population resulting from a given system of breeding shows in the successive generations a series of definite proportions for each of the possible combinations of factors present. The diverse series so obtained, whether from random mating, assortative mating, self-fertilization, or one of the various types of inbreeding, are almost all examples of certain simply derived mathematical series, or of their combinations. It will greatly facilitate the presentation of results and avoid much repetition, if we deal first in a general way with these simple series, giving their laws of formation, and presenting in a table the first twenty terms of each. Our later results may then be presented by designating the terms of the particular series which they give. These fundamental series are shown in table I.

Aside from the series of natural numbers, designated n in the table, the most fundamental series for Mendelian breeding is that of the successive powers of 2; this series is here designated B. Each term is derived from the preceding one by doubling it. The fractions expressing the proportions of the population having any particular combination of factors have as a rule terms of this series as denominators.

[3] The numbers in parentheses refer to the numbered sections or paragraphs in which the formulæ are given later, in the body of the paper.

Closely related to the series of powers of 2 are several series (C, D and E in the table) in which any succeeding term is derived by doubling the preceding one, and adding or subtracting 1 to or from the result. Series C is formed by beginning with 0, doubling each term and adding 1 to give the next term. Each term is 1 less than the corresponding term of B. Series D begins with 2; succeeding terms are obtained by doubling and subtracting 1. Each term is one more than the immediately preceding term of series B. (A series beginning with 1 and formed by doubling each term and subtracting 1 plays a part in breeding, but as all its terms are 1, it does not require elucidation.) Series E is obtained by beginning with 2, doubling and adding 1 to the preceding term. Each of its terms is the difference between a given term of B and the preceding term of D.

Series F is the so-called Fibonacci series; its first two terms are 0 and 1, then each succeeding term is the sum of the two preceding ones. This series is given by the proportions of heterozygotes *Aa* in various systems of inbreeding (q.v.).

Series G is formed by beginning at 0, doubling, then adding or subtracting 1,—alternately for the successive terms,—beginning with addition. Thus, twice 0, plus 1, gives 1; twice 1 minus 1 gives 1; twice 1 plus 1 gives 3, etc. The same series is obtained if we begin with 1, double, and first subtract 1, next time adding 1, etc.; but the first term of the series so obtained will be the second term of G. It may be observed that the difference between any term of G and the corresponding term of B always forms the next term of G. Thus if we subtract each term of G from the corresponding term of B, we obtain anew the series G, though with each term moved one place to the right. Further, the sums of the corresponding terms of G and B give again the series G, though now with each term moved two places to the right. This peculiar series G is characteristic for the results with sex-linked factors.

The remaining series of the table are combinations of those already mentioned (as shown in the second column of the table, headed "How formed"). Thus, series H is given by subtracting each term of F from the corresponding one of G; series I by subtracting from B the corresponding terms of both F and G. Series J is obtained by subtracting from a given term of B the next succeeding term of F, (that is, $J = B_n - F_{n+1}$). In the same way, K is formed by subtracting from any term of B the second succeeding term of F. Series L is another combination of B, F and G. Series K and M are obtained when parents of the type $AA \times Aa$ are inbred.

The table gives the first 20 terms of each series. At the left is, in each case, a letter which will be employed as a designation of the corresponding series. Before each series is given likewise a formula or statement showing, in the first seven series, how any term of the series is derived from the preceding one,—the preceding term being called x. Thus, in series C, $2x + 1$ signifies that any term is formed by doubling the previous term and adding 1. In the remaining six series this formula shows how any given term of the specified series is obtained from the other series. Thus, each term of H is obtained by subtracting the corresponding term of series F from the same term of series G. After the last term of many of the series is given another formula or explanation showing as a rule how any given term of this series may be otherwise designated; thus the fifth term of series C is equal to $2^5 - 1$ (or 31). In some cases certain other information is here given.

Any term of any series is designated by the aid of the letter representing the series, with the corresponding term of the series n as a subscript. Thus, C_1 signifies the first term of C(= 1); F_6 is 8; J_9 is 475; B_0 is 1, etc. In general, if n represents a given number, D_n is the corresponding term of series D, etc. Such expressions as F_{n+2} signify such a term of series F as is found by adding 2 to the given number n; thus if n is 5, F_{n+2} is F_7 (= 13).

In giving results of breeding where these series are involved, there will be given first a general expression showing what terms of the series correspond to the results of n successive breedings of the given type; this will be followed by the first three or four fractions so derived (corresponding to the values 1, 2, 3, 4, etc., of n). For example, under paragraph (45), we find:

$$AA = \frac{J_n}{B_{n+1}};\ \tfrac{1}{4},\ \tfrac{2}{8},\ \tfrac{5}{16} \ldots\ldots \tfrac{1}{2};\ AA_{17} = .490.$$

This signifies that after one inbreeding the proportion of the population having the constitution AA is $\frac{1}{4}$; after two it is $\frac{2}{8}$, after three it is $\frac{5}{16}$ after four $\frac{11}{32}$, and in general after n inbreedings it is the nth term of the series J, divided by the $(n+1)$th term of the series B, so that after 10 inbreedings it is $\frac{J_{10}}{B_{11}}$ or $\frac{935}{2048}$. (See page 63 for further explanation.)

THE RATE OF CHANGE IN THE DIVERSE SYSTEMS OF BREEDING

Working out the concrete results for a single pair of characters, it is found that in most or all the systems of breeding there is a tendency to

approach a constant set of proportions after a large number of generations. In most cases these constant proportions or limits are (theoretically) never actually reached, although the difference between them and the actual proportions may be made less than any designated fraction. Thus, in random mating of sex-linked characters, when the original parents are AA and a — (see (58)), the proportion of males that are A — approaches $\frac{2}{3}$, of those that are a — approaches $\frac{1}{3}$; of the females, AA and Aa each approach $\frac{4}{9}$, while aa approaches $\frac{1}{9}$; but these limiting proportions are never actually reached. In brother by sister mating (29), the proportions of AA and aa each approach $\frac{1}{2}$, while that of Aa approaches 0.

For any particular case, after the formulae have been worked out, as given in the text, the proportions which any particular class approach may be obtained from a table giving once for all the limits approached by the various series of table 1, when divided by the corresponding terms of the series B, — since most of our results take the form of such fractions. For example, in the mating of brother by sister, sex-linked character (69), in the males the proportion of A — is $\frac{G_{n+1}}{B_n}$, and this fraction approaches the value $\frac{2}{3}$; of a the proportion is $\frac{G_n}{B_n}$, which approaches the value $\frac{1}{3}$; in the females AA is $\frac{I_{n+1}}{B_{n+1}}$, which approaches the limit $\frac{2}{3}$, aa is $\frac{H_{n+1}}{B_{n+1}}$, approaching the limit $\frac{1}{3}$; Aa is $\frac{F_{n+1}}{B_n}$, whose limit is 0.

We therefore give in table 2 the limits approached by the fractions obtained when the diverse series are divided by B.

Table 2 includes also certain other data of importance. One needs to know, not only the theoretical limit approached, but also how long a series of breedings is required before the limit is practically attained. In most cases there is at first rapid progress toward the limiting value, and it is attained within say 1 percent after only 6 or 7 generations; thereafter progress is so extremely slow as to be practically negligible. This is the condition already pointed out by PEARL (1913) for certain systems of inbreeding. For example, in self-fertilization, where the original cross was $AA \times Aa$ (see (26)), the proportion of AA gives the series $\frac{E_{n+2}}{B_{n+2}}$ the limit of which is $\frac{3}{4}$, or 75 percent. But after five self-fertilizations AA has already reached the proportion 74.2 percent, so that further breeding in this way has little further effect. In table 2

TABLE 2.—*Limiting values for the various series of fractions given by the division of the terms of the diverse series of table 1 by corresponding terms of series B (of that table); also the earliest term that gives within 1 percent of the limiting value, and the value of this term to three decimal places. The table shows also whether the actual proportion will be above (+) or below (—) the limiting proportion.*

$\frac{C}{B}$ signifies each term of C divided by the same term of B; $\frac{C}{B_{n+1}}$ signifies each term of C divided by the *next later* term of B; $\frac{F}{B_{n-1}}$ signifies each term of F divided by the next preceding term of B, etc.

Series	$\frac{C}{B}$	$\frac{C}{B_{n+1}}$	$\frac{C}{B_{n+2}}$	$\frac{D}{B}$	$\frac{D}{B_{n+1}}$	$\frac{E}{B}$	$\frac{E}{B_{n+1}}$	$\frac{F}{B_{n+1}}$	$\frac{F}{B}$	$\frac{F}{B_{n+1}}$	$\frac{F}{B_{n+2}}$	$\frac{G}{B_{n+1}}$	$\frac{G}{B}$	$\frac{G}{B_{n+2}}$
Limiting value	1	½	¼	½	¼	¾	⅜	0	0	0	0	⅔	⅓	⅙
Term whose value is within 1 percent of limit	7	6	5	7	6	7	6	22	18	15	12	7	6	5
Actual value of this term	.992	.492	.242	.508	.258	.742	.367	.008	.0098	.009	.009	.671	.328	.172
Actual value above (+) or below (—) the limiting value	—	—	—	+	+	—	—	+	+	+	+	±	±	±

Series	$\frac{H}{B}$	$\frac{H}{B_{n+1}}$	$\frac{I}{B}$	$\frac{I}{B_{n+1}}$	$\frac{J}{B}$	$\frac{J}{B_{n+1}}$	$\frac{K}{B}$	$\frac{K}{B_{n+1}}$	$\frac{K}{B_{n+2}}$	$\frac{L}{B}$	$\frac{L}{B_{n+1}}$	$\frac{M}{B_{n+2}}$	$\frac{1}{B}$
Limiting value	⅓	⅙	⅔	⅓	1	½	1	½	¼	⅝	$\frac{5}{12}$	¾	0
Term whose value is within 1 percent of limit	18	15	18	15	21	17	23	20	16	16	13	16	7
Actual value of this term	.324	.157	.657	.324	.992	.490+	.991	.492	.240+	.824	.408	.740+	.008
Actual value above (+) or below (—) the limiting value	—	—	—	—	—	—	—	—	—	—	—	—	+

we give, therefore, the earliest term of the series in which the difference between the final or limiting proportion and the proportion actually attained is less than 1 percent of the total. From this we can readily determine the number of generations required in any system of breeding to bring the actually attained proportion within 1 percent of the theoretical limit.

In most of the series of table 2, the value attained at any term is below the theoretical limit. In the series formed by the division of D and of F by B, the actual value is always above the limit,—the limit being approached from above. In the series formed by dividing G by B, the actual values are alternately below and above the limiting value, the difference decreasing in later terms. These relations are shown in table 2 by use of the signs + and —, as also of ±.

The information summarized in table 2 is given more precisely and fully in the text, under the proper paragraph. After giving the general formula and the first few terms of the series to which it gives rise, the limiting value of the proportions is given, and this is followed by the actual value of the proportions for the smallest number of generations in which the proportion attained is within 1 percent of the limiting value (see explanation on page 63).

DERIVATION OF THE FORMULAE

In the actual working out, the path to most of the formulae has been long and indirect; frequently with many windings and digressions. To give in detail the method of working out each formula would swell the paper beyond all bounds. I am compelled, therefore, in most cases, to content myself with giving the actual formulae, leaving their correctness to the test of time. In a few cases some indications are given of special points requiring attention in the working out. The general method of work is something as follows:

One begins with the formulae of the assumed parents (as $AA \times Aa$), forms the gametes, and mates them (all symbolically of course), obtaining the results for generation 1. One may continue this for five or six generations, getting the proportions of each sort of individuals for each generation. The work is shortened by obtaining at the beginning a formula for transforming the proportions for one generation into those for the next (such a formula as is given in (8)). After obtaining the results for five or six generations, one examines the series of results, and the method of derivation, to discover general relations that may

give a regular series, or be embodied in a formula. Curiously, the study of the actual procedure in deriving results is usually of little assistance in determining the nature of the series or of the general formula. One must, as a rule, take the series of results as finished products, and make an independent study of them, endeavoring by processes of trial to fit them to some series or to some formula. It is here that there is scope for ingenuity; a given series of results may resist for weeks the discovery of the law that unites them. After the system of fundamental series, given in table 1, had been discovered, it was usually found possible to fit the results to some combination of these; but of course the work was three-fourths done before this system was worked out. The series of results as one obtains them are often in perplexing form. For example, in (43), the proportions of *AA* are for five generations successively ½, ⅜ ½, ½, $\frac{5}{8}\frac{5}{4}$. It is difficult to discover the general law underlying such a series, though it turns out that there is one.

After a law or regular series is obtained that fits the first five or six generations, the law is applied to give the results for three or four generations more. These results are then tested by the actual detailed working out (symbolic formation of gametes and their mating, etc.) for these same later generations; if the formula has given the correct results, it is assumed to be a general formula.

More rarely, the formula is obtained by generalizing the actual procedure in the symbolic formation of gametes and their mating according to the given method.

METHOD OF PRESENTATION; DIRECTIONS FOR USE OF THE FORMULAE

The ideal method of presentation of the formulae would be in tables, but this is impracticable, owing to the somewhat complex conditions that must be specified. They will, therefore, be presented in a series of numbered paragraphs. Any single pair of alternative factors will be represented by *A* and *a*, in the three possible combinations *AA*, *aa* and *Aa*. The number of successive times any given sort of breeding has occurred (that is, the number of generations that the system has been followed) will be called *n*. The formulae are designed to show the relative proportions of the population formed by each of the three sorts of individuals (*AA*, *aa* and *Aa*) after any number *n* of breedings of the designated sort (from these the relative proportions of homozygotes and heterozygotes; of dominants and recessives, are of course, at once

obtainable); also the limit which each proportion tends to approach as breeding is continued; and the smallest number of generations that breeding must be continued before the actual proportions come within one percent of the limiting value. The information given will best be understood from an example. In (31), page 72, we find:

$$AA = \frac{M_{n+1}}{B_{n+3}};\ \tfrac{9}{16},\ \tfrac{19}{32},\ \tfrac{49}{64} \ldots\ldots \tfrac{3}{4}.\quad AA_{15} = .7401.$$

Here we have:

1. The general formula for the proportion of the population having the constitution *AA*, after any number *n* of breedings of the sort set forth. This general formula, $\frac{M_{n+1}}{B_{n+3}}$, enables one to find from table 1, page 56, the proportion of *AA* after any particular number of generations (*n*); thus, after 12 such matings($n = 12$), the proportion of *AA* is $\frac{M_{13}}{B_{15}} = \frac{33966}{32768} = .731$.

2. Immediately following the general formula are the first three terms of the series given by the formula; that is, the proportions when *n* is 1, 2, and 3; these are in this case $\frac{9}{16}$, $\frac{19}{32}$, $\frac{49}{64}$.

3. Following these, after a series of dots, is the *limiting value* which these proportions approach. Thus, in this case, after a very large number of inbreedings the proportion of the population that are *AA* would be very near ¾. This limiting value is never actually attained, but the difference between it and the actual value may be made smaller than any designated value.

4. Finally is given, in the form $AA_{15} = .7401$, the lowest number, 15, of inbreedings necessary to bring the actual proportion within 1 percent of the limiting value, together with the actual proportion (.7401) in this generation. Thus, in this case, the formula shows that we must continue inbreeding 15 generations before the individuals *AA* form within 1 percent of the possible ¾ of the population, that at that time they form .7401 of the population, and that if we continue to inbreed we make very little further progress, and can never in this way cause *AA* to form quite .75 of the population.

Similarly, in paragraph (31), page 72, we find given the general formula and the first three terms of the series for the proportions of the population formed by *aa* and *Aa*. We find that *aa* approaches gradually the limit ¼, while *Aa* comes nearer and nearer to 0, although these limits are never quite reached. We find further that after 15 inbreedings *aa* comes within 1 percent of forming ¼ of the population, its actual

proportion at that time being .2401. After 19 generations *Aa* comes within 1 percent of being 0, its actual proportion being then .008.

In some cases the general formula given is the same as that in a preceding paragraph. In such a case the first terms of the series and the limiting values are not repeated; they should be sought in one of the immediately preceding paragraphs. This is the case, for example, in (36); the terms and limit are given in (35).

For some cases the general formula involves other variables besides *n*, so that a concrete series of terms can not be given and no general limit can be assigned. Such are found in (2), (8), etc. These and various other methods of presentation are self-explanatory.

I. Typical factors, not sex-linked

a. *Random mating*

(1) The population at the beginning consists of individuals *AA* and *aa*, in equal numbers. These mate at random (*AA* with *AA* or with *aa*, etc.).

In any later generation the population consists of:

$\frac{1}{4}\ AA + \frac{1}{4}\ aa + \frac{1}{2}\ Aa$
$\frac{1}{2}$ Homozygotes + $\frac{1}{2}$ heterozygotes
$\frac{3}{4}$ Dominants + $\frac{1}{4}$ recessives.

(2) If the proportions of the two kinds are at the beginning not equal, but there are *r AA* to *t aa*, in later generations the population consists of:

$$r^2\ AA + t^2\ aa + 2rt\ Aa$$

The condition given in (1) is only a special case of this, where *r* and *t* are both 1.

(3) If the population at the beginning are all *Aa*, in later generations the constitution is the same as that set forth under (1).

(4) The population at the beginning consists of individuals *AA* and *Aa*, in equal numbers, mating at random (*AA* with *AA* or *Aa*, etc.) In later generations the population consists of:

$\frac{9}{16}\ AA + \frac{1}{16}\ aa + \frac{6}{16}\ Aa$
$\frac{5}{8}$ Homozygotes + $\frac{3}{8}$ heterozygotes
$\frac{15}{16}$ Dominants + $\frac{1}{16}$ recessives

(5) The proportions of AA and Aa at the beginning are not equal, there being r AA to s Aa. In later generations the population is

$$\frac{(s+2r)^2\,AA + s^2aa + 2s(s+2r)\,Aa}{4(s+r)^2}.$$

Case (4) is only a special case of this.

(6) If the population at the beginning is aa and Aa, the results are the same as given under (4) and (5), save that the proportions for aa and AA are interchanged, that for Aa remaining the same.

(7) If a breeding experiment begins with a single pair (AA and aa, or AA and Aa, for example), and these are mated, this first mating *is not a random mating*, but a defined one, and the later conditions specified in (1) and (4) are not reached till after an actual random mating; that is, until the generation f2; they persist thereafter, so long as the matings are random.

(8) The conditions thus far specified are all special cases of the general condition that at the beginning of random mating the population consists of r $AA + t$ $aa + s$ Aa. If r, t and s are diverse and none of them are zero, the constitution of the population is not uniform in later generations, but changes with the number of random matings that occur. If the constitution of any generation n is r $AA + t$ $aa + s$ Aa then the constitution of the next following generation is:

$$AA = (s + 2r)^2.$$
$$aa = (s + 2t)^2.$$
$$Aa = 2(s + 2r)\ (s + 2t).$$

Thus, if at the beginning of random mating the proportions are $3\,AA + 4\,aa + 5\,Aa$, in the next generation the proportions will be:

$$AA = (5 + 2 \times 3)^2 = 11^2 = 121$$
$$aa = (5 + 2 \times 4)^2 = 13^2 = 169$$
$$Aa = 2 \times 11 \times 13 = 286.$$

The proportions for succeeding generations can be found in the same way. I have not been able to obtain a serviceable formula giving directly the proportions for any later generation, without working out the proportions for the intervening generations.

(9) If r and t are equal (whatever the value of s), the later proportions are those given under (1).

(10) If s is zero, the condition set forth under (2) is realized.

(11) If r or t is zero, the condition set forth under (5) or (6) is realized.

(12) If r and t are both zero, the condition given in (3) is realized.

The results for (9), (10), (11) and (12), as given in (1), (2), (5), (6) and (3), flow directly from the relations set forth under (8) when the proper substitutions are made. These relations (8) are the fundamental ones for discovering all the rest.

b. *Assortative mating,—dominants with dominants, recessives with recessives*

If in pure stocks dominants mate with dominants, recessives with recessives, of course the stocks simply remain pure. The problem of how such breeding will result has point only after crossing has occurred, so that heterozygous dominants are present. Therefore, it is necessary to specify in each case what cross or what other sort of mating has occurred before the assortative mating begins.

In such assortative mating after a cross, *AA* mates with *AA* or with *Aa*, while *aa* mates only with *aa*.

(13) The population at the beginning consists of *AA* and *aa* in equal numbers; these mate at random, producing $\frac{1}{4}\ AA + \frac{1}{4}\ aa + \frac{1}{2}\ Aa$; assortative mating then occurs among these progeny.

There are certain pitfalls to be avoided in working out the proportions in later generations. In the generation before assortative mating occurs, the proportion of dominants ($AA + Aa$) is $\frac{3}{4}$, while the recessives are $\frac{1}{4}$. The essential point to keep in mind is that in the next generation the progeny produced by the dominants will be $\frac{3}{4}$ of all the progeny produced, while those from the recessives (which will all be recessive) will be $\frac{1}{4}$. We have then merely to find out what will be the relative proportions of *AA*, *aa* and *Aa* among the progeny of the dominants; to get their proportions of the total we must multiply these proportions from dominants by $\frac{3}{4}$. For *aa* the proportion so obtained (derived from the dominants) must be added to $\frac{1}{4}$, to give the proportion of the total progeny that are recessive. Proceeding in this way, we find that after any number n of successive assortative matings, the proportions of the three classes are:

$$AA = \frac{n+1}{2n+4};\ \tfrac{2}{6},\ \tfrac{3}{8},\ \tfrac{4}{10} \ldots\ldots \tfrac{1}{2}.\quad AA_{48} = .490.$$

$$aa = \frac{n+1}{2n+4}.$$

$$Aa = \frac{1}{n+2};\ \tfrac{1}{3},\ \tfrac{1}{4},\ \tfrac{1}{5} \ldots\ldots 0.\quad Aa_{98} = .010.$$

$$\text{Homozygotes} = \frac{n+1}{n+2}\,;\ \text{heterozygotes} = \frac{1}{n+2}\,.$$

$$\text{Dominants} = \frac{n+3}{2n+4}\,;\ \text{recessives} = \frac{n+1}{2n+4}\,.$$

(14) In the more general case the population at the beginning is $r\ AA$ to $t\ aa$; which mate at random, assortative mating occurring in their progeny.

By (2) the progeny derived from the original parents is

$$r^2AA + 2r\ t\ Aa + t^2aa.$$

Now dominant mates with dominant, recessive with recessive. After n successive assortative matings, the population is:

$$AA = \frac{r(r+nt)}{(r+t)\ (r+nt+t)}\,.$$

$$aa = \frac{(n+1)\ t^2}{(r+t)\ (r+nt+t)}\,.$$

$$Aa = \frac{2rt}{(r+t)\ (r+nt+t)}\,.$$

(15) In the still more general case the population before a given assortative mating, is composed of $r\ AA + t\ aa + s\ Aa$. In the next generation (after the assortative mating), the proportions are:

$$AA = \frac{(2r+s)^2}{4\,(r+s)\,(r+s+t)}\,.$$

$$aa = \frac{s^2+4rt+4st}{4\,(r+s)\,(r+s+t)}\,.$$

$$Aa = \frac{4rs+2s^2}{4\,(r+s)\,(r+s+t)}\,.$$

By continued use of these formulae (substituting at any generation the proportions of AA for r, of aa for t, of Aa for s), the results for any number of generations of assortative matings may be obtained. These relations are the fundamental ones for working out the formulae given in the following paragraphs (as well as in (13)).

(16) The population at the beginning is all Aa; assortative mating occurs. After n such matings:

$$AA = \frac{n}{2n+2}\,;\ \tfrac{1}{4},\ \tfrac{3}{6},\ \tfrac{3}{8}\ \ldots\ldots\ \tfrac{1}{2}\,.\quad AA_{49} = .490\,.$$

$$aa = AA\,.$$

$$Aa = \frac{1}{n+1}\,;\ \tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4}\ \ldots\ldots\ 0\,.\quad Aa_{99} = .010\,.$$

(If the experiment begins by a cross of AA with aa, of course the first assortative mating occurs thus when all are Aa).

(17) The population at the beginning consists of AA and Aa in equal numbers. After n assortative matings (including among these the first mating, where all are dominants), the proportions are as follows:

$$AA = \frac{3n+6}{4n+12}; \text{ thus } \tfrac{9}{16}, \tfrac{12}{20}, \tfrac{15}{24}, \tfrac{18}{28} \ldots\ldots \tfrac{3}{4}. \quad AA_{72} = .740.$$

$$aa = \frac{n}{4n+12}; \text{ thus } \tfrac{1}{16}, \tfrac{2}{20}, \tfrac{3}{24}, \tfrac{4}{28} \ldots\ldots \tfrac{1}{4}. \quad aa_{72} = .240.$$

$$Aa = \frac{6}{4n+12}; \text{ thus } \tfrac{6}{16}, \tfrac{6}{20}, \tfrac{6}{24}, \tfrac{6}{28} \ldots\ldots 0. \quad Aa_{147} = .010.$$

$$\text{Dominants} = \frac{3n+12}{4n+12}; \quad \text{recessives} = \frac{n}{4n+12}.$$

(18) At the beginning we have aa and Aa in equal numbers. If these cross, giving again $aa + Aa$, there is no opportunity for assortative mating. But if instead of crossing, they mate at random (producing $1\,AA + 9\,aa + 6\,Aa$), then assortative mating begins;—after n such matings:

$$AA = \frac{3n+1}{12n+16}; \text{ thus } \tfrac{4}{28}, \tfrac{7}{40}, \tfrac{10}{52} \ldots\ldots \tfrac{1}{4}. \quad AA_{24} = .2401.$$

$$aa = \frac{9(n+1)}{12n+16}; \text{ thus } \tfrac{18}{28}, \tfrac{27}{40}, \tfrac{36}{52} \ldots\ldots \tfrac{3}{4}. \quad aa_{24} = .7401.$$

$$Aa = \frac{6}{12n+16}; \text{ thus } \tfrac{6}{28}, \tfrac{6}{40}, \tfrac{6}{52} \ldots\ldots 0. \quad Aa_{49} = .0099.$$

c. *Selection of the dominants alone*

Only dominants are bred, the recessives being rejected in each generation. In this case, as in assortative mating, some sort of a cross must have occurred at the beginning, otherwise no problem is involved.

(19) The population is at first AA and aa in equal numbers; these mate at random, producing $\frac{1}{4}\,AA + \frac{1}{4}\,aa + \frac{1}{2}\,Aa$; the dominants alone are bred (at random among themselves). After n such breedings, the population is:

$$AA = \frac{(n+1)^2}{(n+2)^2}; \ \tfrac{4}{9}, \tfrac{9}{16}, \tfrac{16}{25} \ldots\ldots 1. \quad AA_{198} = .990+.$$

$$aa = \frac{1}{(n+2)^2}; \ \tfrac{1}{9}, \tfrac{1}{16}, \tfrac{1}{25} \ldots\ldots 0. \quad aa_{8} = .010.$$

$$Aa = \frac{2(n+1)}{(n+2)^2}; \ \tfrac{4}{9}, \tfrac{6}{16}, \tfrac{8}{25} \ldots\ldots 0. \quad Aa_{199} = .0099.$$

(20) In the more general case, the population at first consists of

$r\ AA$ to $t\ aa$, which mate at random (giving according to (2) $r^2AA + t^2aa + 2r\ t\ Aa$); then dominants alone are bred. After n such successive selections, the population is:

$$AA = \frac{(r+n\ t)^2}{(r+(n+1)\ t)^2}. \quad aa = \frac{t^2}{(r+(n+1)\ t)^2}. \quad Aa = \frac{2t\ (r+n\ t)}{(r+(n+1)\ t)^2}.$$

(21) In the still more general case, before a given selective breeding the population is $r\ AA + t\ aa + s\ Aa$.

Of course the $t\ aa$ do not breed at all, and play, therefore, no rôle. After the next selection of dominants:

$$AA = \frac{(2r+s)^2}{4\ (r+s)^2}. \qquad aa = \frac{s^2}{4\ (r+s)^2}. \qquad Aa = \frac{2s\ (2r+s)}{4\ (r+s)^2}.$$

By continuation, the results for any number of generations can be worked out.

(22) If at first AA is crossed with aa, giving Aa, and then dominants alone are bred, the result is the same as in (19), save that any given proportion is reached one generation later. That is,

$$AA = \frac{n^2}{(n+1)^2}. \qquad aa = \frac{1}{(n+1)^2}. \qquad Aa = \frac{2n}{(n+1)^2}.$$

(23) At the beginning aa is crossed with Aa, giving $Aa + aa$; only dominants are bred. This gives the same results as (22).

(24) AA is crossed with Aa, giving $AA + Aa$; then only dominants are bred (at random among themselves). This gives the same series of results as do (19) and (22), but any given result comes one generation sooner than in (19), and two generations sooner than in (22). That is, after n breedings of dominants alone:

$$AA = \frac{(n+2)^2}{(n+3)^2}. \qquad aa = \frac{1}{(n+3)^2}. \qquad Aa = \frac{2n+4}{(n+3)^2}.$$

d. *Self-fertilization*

Some sort of a cross must have occurred before self-fertilization begins, otherwise, of course, the stock remains constant.

(25) Original cross, AA by aa, giving all Aa. Thereafter all breeding is by self-fertilization. In this case AA and aa each give the series C (table 1), while Aa is 2 in every generation. That is, after n self-fertilizations:

$$AA = \frac{C_n}{B_{n+1}}\left(=\frac{2^n-1}{2^{n+1}}\right);\ \tfrac{1}{4},\ \tfrac{3}{8},\ \tfrac{7}{16}\ \ldots\ldots\ \tfrac{1}{2}. \quad AA_6 = .492.$$

$$aa = \frac{C_n}{B_{n+1}}\left(=\frac{2^n-1}{2^{n+1}}\right).$$

$$Aa = \frac{2}{2^{n+1}}\left(=\frac{1}{2^n}\right);\ \tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{1}{8}\ \ldots\ldots\ 0. \quad Aa_7 = .008.$$

(26) Original cross AA by Aa; thereafter self-fertilization. In this case AA gives the series E, beginning at 5; aa gives the series C, beginning at 1, while Aa is always 2. After n self-fertilizations:

$$AA = \frac{E_{n+2}}{B_{n+2}}\left(=\frac{2^{n+1}+2^n-1}{2^{n+2}}\right);\ \tfrac{5}{8},\ \tfrac{11}{16},\ \tfrac{23}{32},\ \tfrac{47}{64}\ \ldots\ldots\ \tfrac{3}{4}.\quad AA_5 = .742.$$

$$aa = \frac{C_n}{B_{n+2}}\left(=\frac{2^{n-1}}{2^{n+2}}\right);\ \tfrac{1}{8},\ \tfrac{3}{16},\ \tfrac{7}{32},\ \tfrac{15}{64}\ \ldots\ldots\ \tfrac{1}{4}.\quad aa_5 = .242.$$

$$Aa = \frac{2}{B_{n+2}}\left(=\frac{1}{B^{n+1}}\right)\left(=\frac{1}{2^{n+1}}\right);\ \tfrac{1}{4},\ \tfrac{1}{8},\ \tfrac{1}{16},\ \tfrac{1}{64}\ \ldots\ldots\ 0.\quad Aa_6 = .008.$$

(27) The population at the beginning shows the typical Mendelian proportions, $\frac{1}{4}\ AA + \frac{1}{4}\ aa + \frac{1}{2}\ Aa$. Now self-fertilization begins and is continued. In this case we obtain the same three series as in (25), but beginning one term farther; that is, after n self-fertilizations:

$$AA = \frac{C_{n+1}}{B_{n+2}}\left(=\frac{2^{n+1}-1}{2^{n+2}}\right);\ \tfrac{3}{8},\ \tfrac{7}{16},\ \tfrac{15}{32}\ \ldots\ldots\ \tfrac{1}{2}.\quad AA_5 = .492.$$

aa = same as AA.

$$Aa = \frac{2}{B_{n+2}}\left(=\frac{2}{2^{n+2}}=\frac{1}{2^{n+1}}\right);\ \tfrac{1}{4},\ \tfrac{1}{8},\ \tfrac{1}{16}\ \ldots\ldots\ 0.\quad Aa_6 = .008.$$

(28) The three cases so far considered are particular instances of the general case that the population consists at the beginning of $r\ AA + t\ aa + s\ Aa$. Then after n self-fertilizations:

$$AA = \frac{r(2^{n+1})+s(2^n-1)}{(r+s+t)\ 2^{n+1}}.\qquad aa = \frac{t(2^{n+1})+s(2^n-1)}{(r+s+t)\ 2^{n+1}}.$$

$$Aa = \frac{2s}{(r+s+t)\ 2^{n+1}}.$$

For any particular constitution of the population at the beginning the results after any number of self-fertilizations will readily be obtained by making the proper substitutions for r, s, t and n in these formulae. For example, if the original population is $2\ AA + 4\ aa + 3\ Aa$, after three self-fertilizations:

$$AA = \tfrac{53}{144}.\qquad aa = \tfrac{91}{144}.\qquad Aa = \tfrac{6}{144}.$$

INBREEDING

Inbreeding may occur according to various diverse systems, and in each case the outcome depends of course on the constitution of the parents originally crossed; or, in the case of an entire population, on the constitution of the population at the time inbreeding begins. In our formulae the number of inbreedings (n) includes only the actual matings of related individuals, not the cross that precedes such matings. The

Fibonacci series (F, table 1) is the key series in most cases of inbreeding.

e. *Brother and sister mating*

When the original cross is either $AA \times aa$, or $AA \times Aa$, or $aa \times Aa$, or $Aa \times Aa$, followed by brother $\times$ sister mating among the progeny, the proportions of heterozygotes Aa form in successive generations a series of fractions of which the terms of the Fibonacci series (F in table 1) are the numerators, while the denominators are certain powers of 2 (forming thus our series B). In the different original matings the series for Aa begins at different points in the F series. Where the original cross is $AA \times aa$ or $Aa \times Aa$, the proportions of AA (or of aa) are $\frac{1}{2}$ of the difference between 1 and the fraction showing the proportion of the heterozygotes Aa. Thus, if the heterozygotes are $\frac{3}{8}$, the proportion for AA is $\frac{1}{2}\ (1 - \frac{3}{8}) = \frac{5}{16}$; the proportion of aa is the same.

Where the original cross is AA by Aa, the proportion for the sum of $(AA + aa)$is $(1 - Aa)$, but $(AA + \frac{1}{2}\ Aa)$ is always equal to three times $(aa + \frac{1}{2}\ Aa)$, so that $AA = 3aa + Aa$. If the original cross is $aa \times Aa$, the proportions are reversed; so that $aa = 3\ AA + Aa$. It results from these relations that the proportions for AA and aa can readily be found when those for Aa are known. When the crosses are $AA \times aa$, or Aa by Aa, the proportions for AA and aa give our series J. When the first cross is AA by Aa, the proportions of AA give the series M, while aa gives the series K. If the first cross is aa by Aa, then aa gives M, while AA gives K. We shall now for reference give the formulae for each of the cases.

(29) Original cross AA by aa. After n brother by sister matings:

$$AA = \frac{J_n}{B_{n+1}};\ \tfrac{1}{4},\ \tfrac{3}{8},\ \tfrac{5}{16},\ \tfrac{11}{32} \ldots\ldots \tfrac{1}{2}.\quad AA_{17} = .490+.$$

$$aa = \frac{J_n}{B_{n+1}}.$$

$$Aa = \frac{F_{n+1}}{B_n};\ \tfrac{1}{2},\ \tfrac{2}{4},\ \tfrac{3}{8},\ \tfrac{5}{16} \ldots\ldots 0.\quad Aa_{21} = .008+.$$

(30) Original cross $Aa \times Aa$. After n inbreedings:

$$AA = \frac{J_{n+1}}{B_{n+2}};\ \tfrac{2}{8},\ \tfrac{5}{16},\ \tfrac{11}{32},\ \tfrac{24}{64} \ldots\ldots \tfrac{1}{2}.\quad AA_{16} = .490+.$$

$$aa = AA.$$

$$Aa = \frac{F_{n+2}}{B_{n+1}};\ \tfrac{2}{4},\ \tfrac{3}{8},\ \tfrac{5}{16},\ \tfrac{8}{32} \ldots\ldots 0.\quad Aa_{20} = .008.$$

(31) Original cross $AA \times Aa$. After n inbreedings:

$$AA = \frac{M_{n+1}}{B_{n+3}};\ \tfrac{9}{16},\ \tfrac{19}{32},\ \tfrac{40}{64} \ldots\ldots\ \tfrac{3}{4}.\quad AA_{15} = .7401.$$

$$aa = \frac{K_{n+1}}{B_{n+3}};\ \tfrac{1}{16},\ \tfrac{3}{32},\ \tfrac{8}{64} \ldots\ldots\ \tfrac{1}{4}.\quad aa_{15} = .2401.$$

$$Aa = \frac{F_{n+3}}{B_{n+2}};\ \tfrac{3}{8},\ \tfrac{5}{16},\ \tfrac{8}{32} \ldots\ldots\ 0.\quad Aa_{19} = .008.$$

(32) Original cross $aa \times Aa$. After n inbreedings:

$$AA = \frac{K_{n+1}}{B_{n+3}};\ \tfrac{1}{16},\ \tfrac{3}{32},\ \tfrac{8}{64} \ldots\ldots\ \tfrac{1}{4}.\quad AA_{15} = .2401.$$

$$aa = \frac{M_{n+1}}{B_{n+3}};\ \tfrac{9}{16},\ \tfrac{19}{32},\ \tfrac{40}{64} \ldots\ldots\ \tfrac{3}{4}.\quad aa_{15} = .7401.$$

$$Aa = \frac{F_{n+3}}{B_{n+2}};\ \tfrac{3}{8},\ \tfrac{5}{16},\ \tfrac{8}{32} \ldots\ldots\ 0.\quad Aa_{19} = .008.$$

(33). In a Mendelian population showing the typical proportions $\frac{1}{4}\ AA + \frac{1}{4}\ aa + \frac{1}{2}\ Aa$, exclusive brother $\times$ sister mating begins at a certain time. After n such inbreedings the results are the same as given in (30).

(34) It may be observed that in all cases the proportions for Aa form a series of fractions in which the numerator of any term is the sum of the numerators of the two preceding terms, while its denominator is double that of the preceding term. One requires therefore but to know the first two terms of any series (as given in preceding paragraphs) in order to write out the entire series as far as he desires.

Similarly, the proportions for AA and aa may be written out by a general rule when the first two terms in any series (as given above) are known. The rule is to *double the numerator and denominator of the preceding term, and add to the numerator the sum of the two last additions so made.* For example, in the series for AA given under (31), to obtain the second numerator 19 from 9, one evidently doubles 9 and adds one; to get 40 from 19, one doubles 19 and adds 2; therefore, to get the numerator of the next term one must double 40 and add $1 + 2 = 3$, giving $\frac{83}{128}$; for the next numerator one doubles 83 and adds $2 + 3 = 5$, giving $\frac{171}{256}$, etc.

The formulae here given are simpler than the one proposed in my paper in the AMERICAN NATURALIST of November, 1914.

f. *Mating of parent by offspring*

The inbreeding of parent by offspring is practicable in higher organisms mainly in the successive breeding back of daughters to the father.

In this way the same individual is used repeatedly as parent in successive generations, as indicated in the diagram (figure 1).

Other systems of breeding parent by offspring are conceivable, and might be carried out in lower organisms, particularly plants. Some of these will be considered for their theoretical interest.

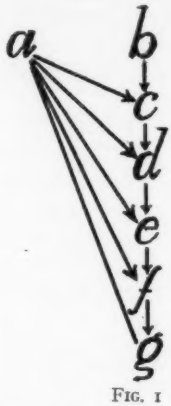

FIG. 1

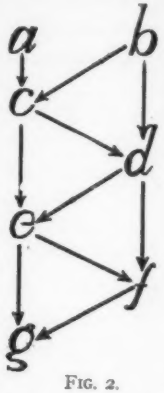

FIG. 2.

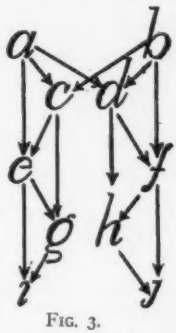

FIG. 3.

FIGURES 1, 2 and 3.—Diagrams illustrating different conceivable systems of mating parents to offspring. The letters represent individuals; the arrows show their parentage.

FIGURE 1.—Continued mating of daughters back to their father. The same father *a* is bred from in each generation.

FIGURE 2.—The alternating system of breeding parent by offspring. Each parent *save one* (*a*) is bred from twice; thus *b* is parent of *c*, and is also mated with *c* to produce *d*; *c* is parent of *d* and *e*, etc. (For simplicity's sake only one offspring is represented in each generation.)

FIGURE 3.—The alternating system of breeding parent by offspring, when both the original parents are represented to the same extent, through breeding half their progeny to one parent, half to the other. (Only one of the offspring from each parent is represented, save that two of the f_1 progeny of *a* and *b* are shown, in order that the family may be divided into two.)

i. Same father bred to his daughters in successive generations (fig. 1).

(35) Original parents *AA* and *aa;* the progeny are always mated back to *AA*. The proportions for *AA* give the series C, while *aa* is 0, and *Aa* is 1. That is, after *n* such inbreedings:

$$AA = \frac{C_n}{B_n}\left(=\frac{2^n-1}{2^n}\right);\ \tfrac{1}{2},\ \tfrac{3}{4},\ \tfrac{7}{8}\ \ldots\ldots\ 1\,.\quad AA_7 = .992\,.$$

$$aa = 0\,.$$

$$Aa = \frac{1}{2^n};\ \tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{1}{8}\ \ldots\ldots\ldots\ldots\ 0\,.\quad Aa_7 = .008\,.$$

(36) Parents $aa \times AA$; progeny mated back to aa. After n inbreedings:

$$aa = \frac{C_n}{B_n}. \qquad AA = 0. \qquad Aa = \frac{1}{2^n}.$$

(37) Parents $Aa \times Aa$; progeny mated back to Aa. This gives the same result as random mating; after n inbreedings:

$$AA = \tfrac{1}{4}. \qquad aa = \tfrac{1}{4}. \qquad Aa = \tfrac{1}{2}.$$

(38 Parents $AA \times Aa$; mated back to AA. After n inbreedings:

$$AA = \frac{C_{n+1}}{B_{n+1}} = \frac{2^{n+1}-1}{2^{n+1}};\ \tfrac{3}{4},\ \tfrac{7}{8},\ \tfrac{9}{16} \ldots\ldots\ 1. \quad AA_6 = .992.$$

$$aa = 0.$$

$$Aa = \frac{1}{2^{n+1}};\ \text{limit } 0. \quad Aa = .008.$$

(39) Parents AA and Aa; progeny mated back to Aa.

Here AA gives the series D, aa gives the series C, while Aa is always ½. That is, after n inbreedings:

$$AA = \frac{D_{n+1}}{B_{n+2}}\left(=\frac{2^n+1}{2^{n+2}}\right);\ \tfrac{3}{8},\ \tfrac{5}{16},\ \tfrac{9}{32} \ldots\ldots\ \tfrac{1}{4}. \quad AA_5 = .258.$$

$$aa = \frac{C_n}{B_{n+2}}\left(=\frac{2^n-1}{2^{n+2}}\right);\ \tfrac{1}{8},\ \tfrac{3}{16},\ \tfrac{7}{16} \ldots\ldots\ \tfrac{1}{4}. \quad aa_5 = .242.$$

$$Aa = \tfrac{1}{2}.$$

(40) Parents $aa \times Aa$; mated to aa. After n inbreedings:

$$aa = \frac{C_{n+1}}{B_{n+1}}. \qquad Aa = \frac{1}{B_{n+1}}. \qquad AA = 0.$$

(41) Parents $aa \times Aa$; mated back to Aa. After n inbreedings:

$$aa = \frac{D_{n+1}}{B_{n+2}}\left(=\frac{2^n+1}{2^{n+2}}\right);\ \text{limit } \tfrac{1}{4}. \qquad AA = \frac{C_n}{B_{n+2}}\left(=\frac{2^n-1}{2^{n+2}}\right);\ \text{limit } \tfrac{1}{4}.$$

$$Aa = \tfrac{1}{2}.$$

(42) In a Mendelian population consisting of ¼ AA + ¼ aa + ½ Aa, at a certain time the female progeny are bred back to their fathers, and this continues. Here AA and aa each gives the series E, beginning at 5, while Aa gives the series D, beginning at 3. That is, after n inbreedings:

$$AA = \frac{E_{n+2}}{B_{n+3}};\ \tfrac{5}{16},\ \tfrac{11}{32},\ \tfrac{23}{64} \ldots\ldots\ \tfrac{3}{8}. \quad AA_4 = .367.$$

$$aa = AA.$$

$$Aa = \frac{D_{n+1}}{B_{n+2}};\ \tfrac{3}{8},\ \tfrac{5}{16},\ \tfrac{9}{32} \ldots\ldots\ \tfrac{1}{4}. \quad Aa_5 = .258.$$

ii. Parent by offspring, alternating system

The system of continuous employment of a single individual as parent with successive generations of its own progeny could not be continued indefinitely, owing to the limited length of life of the individual. It is possible to conceive a system which could be continued indefinitely, by using each parent alternately as the parent of a second generation. That is, each individual would be employed twice, once for breeding with its own parent, once for breeding with its progeny. Such an alternating system is represented in figure 2. Such a system could not well be carried out with higher organisms, since it requires in alternate generations the mating back of the male children to their mother, and the mothers are of course less numerous than the male children. The system could, however, be carried out with certain plants. We shall deal with it without regard to practical considerations, for its theoretical interest. When in this system "parents" are bred to progeny, each "parent" plays (from a statistical or numerical point of view) the same part that the father plays in the continuous system.

(43) The original (unrelated) parents are *AA* and *aa*, giving progeny that are all *Aa*. These *Aa* are bred back to *AA*, and the progeny resulting from this are bred to their parent *Aa*. Thus an alternating system is continued, as illustrated in figure 2.

After mating back to parents has thus occurred n times:

$$AA = \frac{I_{n+1}}{B_{n+1}};\ \tfrac{2}{4},\ \tfrac{3}{8},\ \tfrac{8}{16}\ \ldots\ldots\ \tfrac{2}{3}.\quad AA_{17} = .657.$$

$$aa = \frac{H_{n+1}}{B_{n+1}};\ 0,\ \tfrac{1}{8},\ \tfrac{2}{16}\ \ldots\ldots\ \tfrac{1}{3}.\quad aa_{17} = .3235.$$

$$Aa = \frac{F_{n+1}}{B_{n}};\ \tfrac{1}{2},\ \tfrac{2}{4},\ \tfrac{3}{8}\ \ldots\ldots\ 0.\quad Aa_{21} = .008.$$

(44) The first cross is *aa* by *AA;* the progeny are bred back to *aa*, and the alternating system continues. This gives the same result as (43), save that the proportions of *AA* and *aa* must be interchanged.

(45) It may be observed that by the system given in (43) and (44), and illustrated in figure 2, one of the original individuals (*a* in figure 2) plays a smaller part than all others, being employed as a parent but once, while others are employed twice. It is of interest to determine the results when all parents are equally represented. For this purpose, after the original parents *AA* and *Aa* have produced progeny *Aa*, half of these progeny are mated to the parent *AA*, half to the parent *aa*. The resulting progeny are then mated to their younger parents (*Aa*), and the alternating system continued as before. Figure 3 illustrates this system.

(In working out this system, the correct statistical results are obtained if in each generation we mate *all* the progeny first to one of the two parents, then to the other.)

In this case AA and aa each give the series J, while Aa gives (as in (43) and (44) , the Fibonacci series F. That is, after n inbreedings:

$$AA = \frac{J_n}{B_{n+1}}; \ \tfrac{1}{4}, \ \tfrac{3}{8}, \ \tfrac{5}{16} \ \ldots\ldots \ \tfrac{1}{2}. \quad AA_{17} = .490.$$

$$aa = AA.$$

$$Aa = \frac{F_{n+1}}{B_n}; \ \tfrac{1}{2}, \ \tfrac{3}{4}, \ \tfrac{3}{8} \ \ldots\ldots \ 0. \quad Aa_{21} = .008.$$

(46) Original cross, $AA \times Aa$; progeny bred back to AA, and thenceforth the alternating system of figure 2 is employed. Here AA gives the series L, aa gives the series H, while Aa gives the Fibonacci series (F). After n inbreedings:

$$AA = \frac{L_{n+2}}{B_{n+2}}; \ \tfrac{6}{8}, \ \tfrac{11}{16}, \ \tfrac{24}{32}, \ \tfrac{48}{64}, \ \tfrac{99}{128} \ \ldots\ldots \ \tfrac{5}{6}. \quad AA_{14} = .824.$$

$$aa = \frac{H_{n+1}}{B_{n+2}}; \ 0, \ \tfrac{1}{16}, \ \tfrac{2}{32}, \ \tfrac{6}{64}, \ \tfrac{13}{128} \ \ldots\ldots \ \tfrac{1}{6}. \quad aa_{14} = .157.$$

$$Aa = \frac{F_{n+1}}{B_{n+1}}; \ \tfrac{1}{4}, \ \tfrac{2}{8}, \ \tfrac{3}{16}, \ \tfrac{5}{32}, \ \tfrac{8}{64} \ \ldots\ldots \ 0. \quad Aa_{17} = .0098.$$

(47) Original cross, $AA \times Aa$; progeny bred back to Aa, and thenceforth by the alternating system of figure 2. After n such inbreedings:

$$AA = \frac{I_{n+2}}{B_{n+2}}; \ \tfrac{3}{8}, \ \tfrac{8}{16}, \ \tfrac{16}{32} \ \ldots\ldots \ \tfrac{2}{3}. \quad AA_{16} = .657.$$

$$aa = \frac{H_{n+2}}{B_{n+2}}; \ \tfrac{1}{8}, \ \tfrac{2}{16}, \ \tfrac{6}{32} \ \ldots\ldots \ \tfrac{1}{3}. \quad aa_{16} = .323.$$

$$Aa = \frac{F_{n+2}}{B_{n+1}}; \ \tfrac{2}{4}, \ \tfrac{3}{8}, \ \tfrac{5}{16} \ \ldots\ldots \ 0. \quad Aa_{20} = .008.$$

(48) If with parents $AA \times Aa$ we make the part played by the two original parents equal, by following the scheme represented in figure 3, we obtain after n inbreedings the following results:

$$AA = \frac{M_{n+1}}{B_{n+3}}; \ \tfrac{9}{16}, \ \tfrac{19}{32}, \ \tfrac{40}{64} \ \ldots\ldots \ \tfrac{3}{4}. \quad AA_{15} = .7401.$$

$$aa = \frac{K_{n+1}}{B_{n+3}}; \ \tfrac{1}{16}, \ \tfrac{3}{32}, \ \tfrac{8}{64} \ \ldots\ldots \ \tfrac{1}{4}. \quad aa_{15} = .2401.$$

$$Aa = \frac{F_{n+3}}{B_{n+2}}; \ \tfrac{3}{8}, \ \tfrac{5}{16}, \ \tfrac{8}{32} \ \ldots\ldots \ 0. \quad Aa_{19} = .008.$$

(49) Parents $aa \times Aa$; alternating system of figure 2, beginning with aa. After n inbreedings the proportions are as in (46), with those for AA and aa interchanged. That is:

$$aa = \frac{L_{n+2}}{B_{n+2}}. \qquad AA = \frac{H_{n+1}}{B_{n+2}}. \qquad Aa = \frac{F_{n+1}}{B_{n+1}}.$$

(50) Parents $aa \times Aa$; alternating system of figure 2, beginning with Aa. After n such inbreedings the proportions are as in (47), with those for AA and aa interchanged.

(51) Parents $aa \times Aa$; equalized alternating system of figure 3. After n such inbreedings the proportions are as in (48), with those for AA and aa interchanged.

(52) Original parents $Aa \times Aa$. In this case the same results are reached whether we employ the system shown in figure 2, or that shown in figure 3. After n inbreedings:

$$AA = \frac{J_{n+1}}{B_{n+2}}; \quad \tfrac{2}{8}, \ \tfrac{5}{16}, \ \tfrac{12}{32} \ \ldots\ldots \ \tfrac{1}{2}. \quad AA_{16} = .490.$$

$$aa = \frac{J_{n+1}}{B_{n+2}}.$$

$$Aa = \frac{F_{n+2}}{B_{n+1}}; \quad \tfrac{2}{4}, \ \tfrac{3}{8}, \ \tfrac{5}{16} \ \ldots\ldots \ 0. \quad Aa_{20} = .008.$$

iii. Half the progeny bred to one parent, half to the other

It is theoretically interesting to examine what results would follow if half the progeny were bred each time to one of the parents, half to the other. This system could be carried out in plants in which vegetative reproduction by cuttings is possible. The daughters would be bred, as usual, to the father. From the mother-plant would be produced by cutting as many plants as there are sons, and one of these would be bred to each son. The relative numbers of progeny in the next generation would thus depend on the relative numbers of different kinds of their *younger* parents, as in the mating of father by daughters.

This system gives a number of peculiar series of results, seemingly not reducible to simple general formulae, so that I shall have to content myself with giving rules for obtaining any term of the series when preceding terms are known, together with a number of the earlier terms in each series.

(53) Original parents $AA \times aa$. Half the progeny are mated to one parent, half to the other, and this system is continued.

What we desire is to obtain the three series of fractions giving the proportions of AA, aa and Aa, for successive values of n. The results for Aa give the key to those for the others, so that Aa will be dealt with first. The rule obtained is such as to require that before its application the first two terms shall be known; these for Aa are: after 1

inbreeding $\frac{3}{4}$; after 2 inbreedings $\frac{6}{16}$. We desire then to obtain a rule for finding the term in the series for any value of n, when the terms for $n - 1$ and $n - 2$ are known.

To obtain the nth term of the series:

For *Aa:* If n is odd, double the numerator and denominator of the term for $n - 1$, and from the resulting numerator subtract the numerator of the term for $n - 2$. Thus, since the first term is $\frac{2}{4}$ and the second $\frac{6}{16}$, the third term is $\frac{12-2}{32} = \frac{10}{32}$.

If n is even, multiply by *four* the numerator and denominator of the term for $n - 1$, and from the resulting numerator subtract the numerator of the term for $n - 2$. Thus, as term 2 is $\frac{6}{16}$ and term 3 is $\frac{10}{32}$ term 4 will be $\frac{40-6}{128} = \frac{34}{128}$.

(The fractions expressing the values must not be reduced to lower terms, otherwise incorrect results are reached, or the numerators themselves become fractional.)

For *AA* or *aa* (the values are the same for both):

If n is odd, double numerator and denominator of the term for $n - 1$, and to the resulting numerator add $\frac{1}{2}$ the numerator of *Aa* for $n - 2$. Thus, term 1 is $\frac{1}{4}$; term 2 is $\frac{5}{16}$; therefore term 3 is $\frac{10+1}{32} = \frac{11}{32}$ (since term 1 for *Aa* is $\frac{2}{4}$).

If n is even, multiply by 4 the numerator and denominator of the term for $n - 1$, and add to the resulting numerator $\frac{1}{2}$ the numerator of *Aa* for $n - 2$. Thus, since term 3 is $\frac{11}{32}$, term 4 will be $\frac{44+3}{128} = \frac{47}{128}$.

The first ten terms of the series are thus:

$n =$	1	2	3	4	5	6	7	8	9	10
Aa	2	6	10	34	58	198	338	1154	1970	6726
AA	1	5	11	47	99	413	855	3519	7207	29405
aa	1	5	11	47	99	413	855	3519	7207	29405
Total	4	16	32	128	256	1024	2048	8192	16384	65536

(It may be noted that five of these terms were worked out before the rule was obtained; that three more were then obtained through the rule and verified by working out in detail; while the last two are based on the rule alone.)

The limiting value for *AA* and for *aa* is apparently $\frac{1}{2}$; for *Aa*, 0; but I have not carried the results far enough to make this certain.

(54) Original parents *Aa* × *Aa;* half the progeny mated to one parent, half to the other.

The rules here are the same as in (53), save that the relations of odd and even are interchanged; also the series begin differently (that is, to

obtain an odd-numbered term we multiply by 4, etc., to get an even-numbered term we multiply by 2, etc.). The series of proportions are:

$n =$	1	2	3	4	5	6	7	8	9	10
Aa	8	14	48	82	280	478	1632	2786	9512	16238
AA	4	9	40	87	372	785	3280	6799	28012	57417
aa	4	9	40	87	372	785	3280	6799	28012	57417
Total	16	32	128	256	1024	2048	8192	16384	65536	131072

(55) Original parents $AA \times Aa$; half the progeny mated regularly to each parent.

The rules here are the same as in (54), though the series begin differently and give diverse results. For *Aa*, if n is odd, multiply by 4 the numerator and denominator of term $n - 1$, and subtract numerator of $n - 2$; if n is even, multiply by 2, and subtract numerator of $n - 2$, etc. The first ten terms of the resulting series are:

$n =$	1	2	3	4	5	6	7	8	9	10
Aa	6	10	34	58	198	338	1154	1970	6726	11482
AA	9	19	79	163	669	1367	5567	11303	45789	92563
aa	1	3	15	35	157	343	1471	3111	13021	27027
Total	16	32	128	256	1024	2048	8192	16384	65536	131072

It will be observed that here, as in (31), $(AA + \frac{1}{2}\, Aa) = 3\ (aa + \frac{1}{2}\, Aa)$.

If the original parents were *aa* and *Aa*, doubtless the results would be the same, save with the proportions for *AA* and *aa* interchanged.

II. Sex-linked factors

The peculiarities of sex-linked factors are, for present purposes: (1) The males consist of but two classes with respect to such factors, while the females fall into the usual three classes. (2) The males are never double dominants; they never show the constitution *AA*. Two different methods of conceiving the constitution of the males are extant, resulting in two diverse methods of representing them. For our purposes both methods have the same results, but one is more convenient than the other. According to the absolute "presence and absence" theory, the two classes of males are to be designated *Aa* and *aa*; one is a heterozygote, the other a pure recessive, while no pure dominant exists. According to Morgan's results, the male is conceived to have but one factor, for either dominant or recessive, so that the two classes of males are to be represented as *A* — and *a* —. Both systems of designation give (of course) in later generations the same proportions of the various classes of males and females. But the system which represents differ-

ently the males and females is the more convenient in computing and representing results. We shall represent the two classes of males by A — and a —, the three classes of females by AA, aa and Aa. Anyone who desires to do so may substitute a for the dash in the formula for the males. The only difference produced will be to make it, in working out formulae, difficult to determine the proportions of males and females having any particular constitution, and as a consequence, to determine what later matings should be made.

The characteristic feature in the results with sex-linked characters is the frequent appearance of the series G, which is of such a nature (as will be seen) that the proportions of individuals having a given constitution do not change continuously in one direction, but show successive slight increases and decreases, though with a general trend in one direction.

In the formulae for sex-linked characters the proportions of the males and of the females will be given separately. In all cases the number of males and females is the same.

a. *Random mating*

(56) The population at the beginning consists of two biotypes in equal numbers, each with equal numbers of the two sexes; that is, on the one hand of AA and A —, on the other of aa and a —. These mate at random (any male with any female).

In any later generation the population is:

Males: $\frac{1}{2}\ A\ — + \frac{1}{2}\ a\ —$

Females: $\frac{1}{4}\ AA + \frac{1}{4}\ aa + \frac{1}{2}\ Aa.$

Of the total population $\frac{5}{8}$ are dominants, $\frac{3}{8}$ recessives.

(57) In the more general case the proportions at the beginning are: $r\ AA$ and $r\ A$ —; $t\ aa$ and $t\ a$ — (there being thus two biotypes differing in number, but with equal numbers of males and females in each). After random mating for any number of generations the proportions are:

$$\text{Males: } A— = \frac{r}{r+t}. \qquad a— = \frac{t}{r+t}.$$

$$\text{Females: } AA = \frac{r^2}{(r+t)^2}. \qquad aa = \frac{t^2}{(r+t)^2}. \qquad Aa = \frac{2rt}{(r+t)^2}.$$

(58) The population at the beginning is AA and a — (two biotypes in equal numbers, but all the males belonging to one, all the females to the other). After the first cross all breeding is by random mating.

This case gives certain curious series of proportions as the number

of successive random matings increases. The two sorts of males give the series G (table 1, page 56), while the three classes of females give series formed by products of succeeding terms of G, or compounds of these with B. That is, after n generations of random mating (not including in n the first cross):

Males:—

$$A\text{—} = \frac{G_{n+1}}{B_n};\ \tfrac{1}{2},\ \tfrac{3}{4},\ \tfrac{5}{8},\ \tfrac{11}{16},\ \text{etc.}\ \ldots\ldots\ \tfrac{2}{3}.\quad AA_6 = .672.$$

$$a\text{—} = \frac{G_n}{B_n};\ \tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{3}{8},\ \tfrac{5}{16},\ \text{etc.}\ \ldots\ldots\ \tfrac{1}{3}.\quad aa_6 = .328.$$

Females:—

$$AA = \frac{G_n \times G_{n+1}}{B_{2n-1}};\ \tfrac{1}{2},\ \tfrac{3}{8},\ \tfrac{15}{32},\ \tfrac{55}{128},\ \tfrac{231}{512},\ \tfrac{903}{2048}\ \ldots\ldots\ \tfrac{4}{9}.\quad AA_5 = .451.$$

$$aa = \frac{G_n \times G_{n-1}}{B_{2n-1}};\ 0,\ \tfrac{1}{8},\ \tfrac{3}{32},\ \tfrac{15}{128},\ \tfrac{55}{512},\ \tfrac{231}{2048}\ \ldots\ldots\ \tfrac{1}{9}.\quad aa_5 = .107.$$

$$Aa = 1 - AA - aa;\ \tfrac{1}{2},\ \tfrac{4}{8},\ \tfrac{14}{32},\ \tfrac{58}{128},\ \tfrac{236}{512},\ \tfrac{914}{2048}\ \ldots\ldots\ \tfrac{4}{9}.\quad Aa_3 = .437.$$

The above series given by the different constitutions present some interesting points. Any term n of the values for A — is given by the sum of n terms of the following well-known series:

$$\tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{8} + \tfrac{1}{16} - \tfrac{1}{32} + \ldots\ldots \text{etc.}$$

If we discontinue this series after any number n of terms, and take its sum to that point, we shall have the proportion of A— for n random matings.

Similarly, the series of values for a — are given by the sums of the following series, stopped at any point:

$$\tfrac{1}{2} - \tfrac{1}{4} + \tfrac{1}{8} - \tfrac{1}{16} + \tfrac{1}{32} - \ldots\ldots \text{etc.}$$

In both cases the signs of the terms (after the first) alternate, and each successive denominator is double the preceding one.

In the females the values of the different terms are given by the sums of still more curious series. The nature of these series will best be understood by taking a concrete case. After 5 random matings ($n = 5$), the values for AA, aa and Aa are given by the sums of the following series:

$$aa_5 = \tfrac{0}{2} + \tfrac{1}{4} - \tfrac{2}{8} + \tfrac{3}{16} - \tfrac{4}{32} + \tfrac{4}{64} - \tfrac{3}{128} + \tfrac{2}{256} - \tfrac{1}{512} = \tfrac{55}{512}.$$

$$AA_5 = \tfrac{1}{2}[+\tfrac{0}{4} - \tfrac{1}{8} + \tfrac{5}{16} - \tfrac{8}{32} + \tfrac{4}{64} - \tfrac{8}{128} + \tfrac{2}{256} - \tfrac{1}{512}] = \tfrac{231}{512}.$$

$$Aa_5 = \tfrac{1}{2}[-\tfrac{1}{4} + \tfrac{3}{8} - \tfrac{5}{16} + \tfrac{7}{32} - \tfrac{8}{64} + \tfrac{6}{128} - \tfrac{4}{256} + \tfrac{2}{512}] = \tfrac{236}{512}.$$

These series for any value of n can be written by taking note of the following facts:

For *aa:* (1) The first term is $\frac{0}{2}$; (2) The total number of terms is

$2n - 1$. (3) The terms after the first alternate in sign, the sign of the second term being +. (4) The numerators increase successively by 1 up to the number $n - 1$; the series of numerators is then repeated in reverse order, to 1. (5) The denominator is doubled for each successive term.

For *AA:* (1) The value is equal to ½, with the series within the brackets; (2) The series within the brackets begins with $+\frac{2}{4}$; (3) The number of terms in the brackets is $2n - 2$. (4) The terms of this series alternate in sign. (5) The numerators of its terms are increased successively by 1, up to the number $n - 1$, then decrease by 1, to unity. (6) The denominator is doubled for each successive term.

For *Aa:* (1) The value is ½, with the series in brackets. (2) The first term of the series in brackets is $-\frac{1}{4}$. (3) The number of terms in the brackets is $2n - 2$. (4) The terms of this series alternate in sign. (5) The successive numerators are the successive odd numbers up to the number $(2n - 3)$, followed by the next higher even number, then by the descending series of even numbers, to 2. (6) The denominator is doubled for each successive term.

(59) If the population at the beginning is *aa* and *A* —, in equal numbers, the results are the same as in (58), save that:

The values for *A* — and *a* — are to be interchanged;
The values for *AA* and *aa* are to be interchanged;
The values for *Aa* remain the same.

b. *Assortative mating*

(60) The population consists of the progeny of *AA*, *aa*, *A* — and *a* —, which were present in equal numbers and have mated at random. That is, the population consists (by (56)) of: females, ¼ *AA* + ¼ *aa* + ½ *Aa;* males, ½ *A* —, ½ *a* —. These thenceforth mate assortatively; the dominants *AA* and *Aa* mate only with *A* —, while the recessive *aa* mates only with *a* —.

In the case of sex-linked factors the number of dominant males is less than that of dominant females. It must be assumed that one male may fertilize several females, so that no proportion of any particular class of females goes unfertilized on account of relative scarcity of males. The proportions of progeny produced by dominants on the one hand, by recessives on the other, will therefore depend entirely on the relative proportions of dominant and recessive *female* parents (provided that, as is always the case, some considerable number of males of each

class exist.) In working out the proportions in later generations, it must be assumed that all female dominants are fertilized by A —, all female recessives by a —. In this case A — and AA give the series E; a — gives the series D. After n assortative matings, the proportions are:

Males: $A- = \frac{E_{n+1}}{B_{n+1}}$; $\frac{3}{4}, \frac{5}{8}, \frac{11}{16} \ldots\ldots \frac{3}{4}$. $A-_6 = .742$.

$a- = \frac{D_n}{B_{n+1}}$; $\frac{3}{4}, \frac{3}{8}, \frac{5}{16} \ldots\ldots \frac{1}{4}$. $a-_6 = .258$.

Females: $AA = \frac{E_{n+1}}{B_{n+1}}$; $\frac{3}{4}, \frac{5}{8}, \frac{11}{16} \ldots\ldots \frac{3}{4}$. $AA_6 = .742$.

$aa = \frac{1}{4}$.

$Aa = \frac{1}{B_{n+1}}$; $\frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots\ldots 0$. $Aa_6 = .008$.

(61) At the beginning, AA is crossed with a —, the progeny mate assortatively. The progeny are of course $Aa + A$ —, males and females in equal number.

In this method of mating it turns out that the recessives do not propagate at all, since no female recessives (aa) are formed, and the female heterozygotes of course mate with male dominants. If the females reproduce in proportion to the numbers of the two classes that exist (AA and Aa), then after n such assortative matings the population in the nth generation is:

Males: $A- = \frac{2^n - 1}{2^n}$; $a- = \frac{1}{2^n}$.

Females: $AA = \frac{2^n - 1}{2^n}$, limit 1. $AA_7 = .992$.

$Aa = \frac{1}{2^n}$; limit 0. $Aa_7 = 008$.

The same result is reached if we make at the beginning a reciprocal cross, AA by a — and aa by A —, the progeny mating assortatively.

(62) The original cross is aa by A —. This gives progeny $Aa + a$ —, which gives no opportunity for assortative mating. If these are crossed again, they give $Aa + aa + A - + a$ —, in which assortative mating may occur. After n successive assortative matings the population is:

Males: $A- = \frac{C_n}{B_{n+1}}\left(= \frac{2^n - 1}{2^{n+1}}\right)$; $\frac{1}{4}, \frac{3}{8}, \frac{7}{16} \ldots\ldots \frac{1}{2}$. $A-_6 = .492$.

$a- = \frac{D_{n+1}}{B_{n+1}}\left(= \frac{2^n + 1}{2^{n+1}}\right)$; $\frac{3}{4}, \frac{5}{8}, \frac{9}{16} \ldots\ldots \frac{1}{2}$. $a-_6 = .508$.

Females: $AA = \frac{C_n}{B_{n+1}} \left(= \frac{2^n - 1}{2^{n+1}}\right)$.

$aa = \frac{1}{2}$.

$Aa = \frac{1}{2^{n+1}}$.

c. *Dominants alone selected*

If our original cross is either AA by a —, or aa by A — (followed by crossing of their progeny Aa and a —); or we have the double reciprocal crosses AA by a — and aa by A —; or it is Aa by A —; or Aa by a —, the same series of results is obtained after n generations of breeding from dominants alone, but in some cases a given result is reached earlier than in others, thus requiring in the formulae a slightly different relation to n. I will therefore list the formulae for each case.

Original cross. After n selections of dominant alone:

(63) AA by a —.
(64) aa by A — (giving Aa and a —, which are bred together).
(65) Reciprocal crosses, AA by a — and aa by A —.
(66) Aa by a —.

AA or A — $= \frac{C_n}{B_{n+1}}$, limit 1.

Aa or a — $= \frac{1}{B_{n+1}}$, limit 0.

$aa = 0$.

(67) Aa by A —. AA or A — $= \frac{C_{n+1}}{B_{n+2}}$. Aa or a — $= \frac{1}{B_{n+2}}$.

$aa = 0$.

(68) Original population AA, aa, A — and a —, in equal numbers, mating at random so as to give: males ½ A — + ½ a —; females ¼ AA + ¼ aa + ½ Aa. Now begins selection of dominants alone. After n such selections:

AA or A— $= \frac{E_{n+1}}{3\ B_{n-1}}$; $\frac{3}{8}$, $\frac{5}{8}$, $\frac{11}{12}$, $\frac{23}{24}$ ..... 1. AA_7 or A—$_7$ = .995.

Aa or a— $= \frac{1}{3\ B_{n-1}}$; $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$ ..... 0. Aa_7 or a—$_7$ = .005

$aa = 0$.

INBREEDING AND SEX-LINKED FACTORS

d. *Mating of brother by sister*

(69) Original cross, $AA \times a$ —; brother and sister mating then continues for n generations.

Males: $A— = \frac{G_{n+1}}{B_n}$; $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ ..... $\frac{2}{3}$. $A—_6 = .672$.

$a— = \frac{G_n}{B_n}$; $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$ ..... $\frac{1}{3}$. $a—_6 = .328$.

Females: $AA = \frac{I_{n+1}}{B_{n+1}}$; $\frac{2}{4}$, $\frac{3}{8}$, $\frac{8}{16}$, $\frac{16}{32}$ ..... $\frac{2}{3}$. $AA_{17} = .657$.

$aa = \frac{H_{n+1}}{B_{n+1}}$; 0, $\frac{1}{8}$, $\frac{2}{16}$, $\frac{6}{32}$ ..... $\frac{1}{3}$. $aa_{17} = .324$.

$Aa = \frac{F_{n+1}}{B_n}$; $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$, $\frac{5}{16}$ ..... 0. $Aa_{21} = .008$.

(70) The original cross is aa by A —.

The results here are the same as in the last (69) save that:

The values for AA and aa are interchanged.
The values for A — and a — are interchanged.
The values for Aa are the same as in (69).

(71) At the beginning there are reciprocal crosses, AA by a —, and aa by A —; thereupon brother by sister matings occur. After n such matings, the population is:

Males: $A— = \frac{1}{2}$. $a— = \frac{1}{2}$.

Females: $AA = \frac{J_n}{B_{n+1}}$; thus $\frac{1}{4}$, $\frac{2}{8}$, $\frac{5}{16}$ ..... $\frac{1}{2}$. $AA_{17} = .490+$

$aa = \frac{J_n}{B_{n+1}}$.

$Aa = \frac{F_{n+1}}{B_n} = \frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$ ..... 0. $Aa_{21} = .008$.

(72) The original cross is Aa by A —.

This gives the same result as (69), save that any given set of proportions is reached one generation sooner. That is, after n inbreedings:

Males: $A— = \frac{G_{n+2}}{B_{n+1}}$. $a— = \frac{G_{n+1}}{B_{n+1}}$.

Females: $AA = \frac{I_{n+2}}{B_{n+2}}$. $aa = \frac{H_{n+2}}{B_{n+2}}$. $Aa = \frac{F_{n+2}}{B_{n+1}}$.

(73) The original cross is Aa by a —.

Results as in (72) save that:

The proportions for A — and a — are interchanged.
The proportions for AA and aa are interchanged.
The values for Aa are the same.

e. *Mating of parent by offspring*

i. Same parent (father) used successively

(74). The parents are A — and aa; after this cross the female progeny are bred back to their father, this continuing for n generations.

Of the males:

$$A- = \frac{C_n}{B_n}\left(\text{or } \frac{2^n-1}{2^n}\right); \text{ limit 1.} \quad A-_7 = .992.$$

$$a- = \frac{1}{B_n}; \text{ limit 0.} \quad a-_7 = .008.$$

Of the females:

$$AA = \frac{C_n}{B_n}\left(\text{or } \frac{2^n-1}{2^n}\right). \qquad aa = 0. \qquad Aa = \frac{1}{B_n}.$$

(75) The parents are A — and Aa; the female progeny are bred to their father for n successive generations.

Of the males:

$$A- = \frac{C_{n+1}}{B_{n+1}}; \text{ limit 1.} \quad A-_6 = .992. \quad a- = \frac{1}{B_{n+1}}; \text{ limit 0.} \quad a-_6 = .008.$$

Of the females:

$$AA = \frac{C_{n+1}}{B_{n+1}}, \left(\text{or } \frac{2^{n+1}-1}{2^{n+1}}\right). \qquad aa = 0. \qquad Aa = \frac{1}{B_{n+1}}.$$

(76) The parents are a — and AA; after the cross the female progeny are bred to their father.

This gives the same result as (74), save that the results for A — are to be interchanged with those for a —; also the results for AA with those for aa, while Aa is unaltered.

(77) Parents a — and Aa; successive female progeny bred to father.

Results as in (75), with A — and a — interchanged; also AA and aa are interchanged.

ii. All female progeny mated to father, all male progeny to mother

It may be of theoretical interest to examine further what would be the result with sex-linked characters if all the daughters were mated to their father, all the sons to their mother. Such a system could be carried out with bisexual plants that could be propagated by vegetative cuttings. The mating of the daughters with the father would present no difficulty. For the mating of sons with their mother, cuttings would be taken of the mother plant, in number equal to the sons, and each son mated to one of these.

(78) Original cross AA by a —; daughters thereafter mated to their fathers, sons to their mothers.

Here in the case of the males, A — gives the odd terms of the series G, beginning with $G_3 = 3$, while a — gives the even terms of this series, beginning with $G_2 = 1$. The females give other modifications of this same series. After n inbreedings:

Males:

$$A- = \frac{G_{2n+1}}{B_{2n}};\ \tfrac{3}{4},\ \tfrac{11}{16},\ \tfrac{43}{64},\ \tfrac{171}{256} \ldots\ldots\ \tfrac{2}{3}. \qquad A-_8 = .672.$$

$$a- = \frac{G_{2n}}{B_{2n}};\ \tfrac{1}{4},\ \tfrac{5}{16},\ \tfrac{21}{64},\ \tfrac{85}{256} \ldots\ldots\ \tfrac{2}{3}. \qquad a-_8 = .328.$$

Females:

$$AA = \frac{G_{2n+2} - 3^{n-1}}{B_{2n+1}};\ \tfrac{4}{8},\ \tfrac{18}{32},\ \tfrac{76}{128},\ \tfrac{314}{512} \ldots\ldots\ \tfrac{3}{8}. \qquad AA_{10} = .657.$$

$$aa = \frac{G_{2n+1} - 3^{n-1}}{B_{2n+1}};\ \tfrac{2}{8},\ \tfrac{8}{32},\ \tfrac{34}{128},\ \tfrac{144}{512} \ldots\ldots\ \tfrac{1}{8}. \qquad aa_{10} = .324.$$

$$Aa = \frac{3^{n-1}}{B_{2n}};\ \tfrac{1}{4},\ \tfrac{3}{16},\ \tfrac{9}{64},\ \tfrac{27}{256} \ldots\ldots\ 0. \qquad Aa_{18} = .008.$$

(79) The parents are aa and A —; breeding as in (78).

Results as in (78), save that the proportions for A — and a — must be interchanged; also the proportions for AA and aa must be interchanged. Aa remains the same.

(80) At the beginning we make the reciprocal crosses AA by a — and aa by A —; thereafter daughters are mated to fathers, sons to mothers. After n such matings:

Males: $A- = \frac{1}{2}$. $\quad a- = \frac{1}{2}$.

Females: $$AA = \frac{B_{2n} - 3^{n-1}}{B_{2n+1}};\ \tfrac{3}{8},\ \tfrac{13}{32},\ \tfrac{55}{128},\ \tfrac{223}{512} \ldots\ldots\ \tfrac{1}{2}. \qquad AA_{10} = .491.$$

$$aa = AA.$$

$$Aa = \frac{3^{n-1}}{B_{2n}};\ \tfrac{1}{4},\ \tfrac{3}{16},\ \tfrac{9}{64},\ \tfrac{27}{256} \ldots\ldots\ 0. \qquad Aa_{18} = .008.$$

(81) Original parents Aa by A —; thereafter daughters are mated to fathers, sons to mothers. After n such matings:

Males:

$$A- = \frac{G_{2n+2}}{B_{2n+1}};\ \tfrac{5}{8},\ \tfrac{21}{32},\ \tfrac{85}{128},\ \tfrac{341}{512} \ldots\ldots\ \tfrac{2}{3}. \qquad A-_8 = .664.$$

$$a- = \frac{G_{2n+1}}{B_{2n+1}};\ \tfrac{3}{8},\ \tfrac{11}{32},\ \tfrac{43}{128},\ \tfrac{171}{512} \ldots\ldots\ \tfrac{1}{3}. \qquad a-_8 = .336.$$

Females:

$$AA = \frac{G_{2n+3} - 3^n}{B_{2n+2}};\ \tfrac{8}{16},\ \tfrac{34}{64},\ \tfrac{144}{256},\ \tfrac{602}{1024} \ldots\ldots \tfrac{2}{3}. \qquad AA_{12} = .659.$$

$$aa = \frac{G_{2n+2} - 3^n}{B_{2n+2}};\ \tfrac{2}{16},\ \tfrac{12}{64},\ \tfrac{58}{256},\ \tfrac{200}{1024} \ldots\ldots \tfrac{1}{3}. \qquad aa_{12} = .326.$$

$$Aa = \frac{3^n}{B_{2n+1}};\ \tfrac{8}{8},\ \tfrac{9}{32},\ \tfrac{27}{128},\ \tfrac{81}{512} \ldots\ldots 0. \qquad Aa_{14} = .009.$$

(82) Parents Aa by a —; breeding as in (81).

Results as in (81), but the proportions of A — and a — must be interchanged, as must also those for AA and aa, while Aa remains unaltered.

The results when two or more factors, independent or linked, are considered, will be dealt with in a later paper.

SUMMARY

This paper gives formulae for finding in any generation the results of continued breeding by a given system, with respect to a single pair of alternative characters. Sex-linked characters and typical characters are dealt with separately. Formulae are given for the results of: random mating; assortative mating; selection of dominants; self-fertilization; inbreeding of brother by sister and of parent by offspring (several systems). In each case the diverse results obtained by beginning with different parental combinations are given.

It is shown that the results in successive generations form fractions such as may be obtained by compounding in various ways several well known arithmetical series. The first twenty terms are given of fourteen such series; the formulae show how the terms of these series are to be compounded in order to give the results of a particular type of breeding for any designated number of generations.

In addition to the general formulae, there are given the first three terms of each series of fractions obtained by given methods of breeding, and the limiting value toward which the series tend. These limiting values show the approximate proportion of the population that will have a given constitution after breeding by a given system for an indefinitely large number of generations. Furthermore, the paper gives the least number of generations of breeding by a given system required to approach within one percent of the final limit. This number is usually relatively small; after it is passed, continued breeding by the same system has little effect in changing the composition of the population.

The results of the combinations given by diverse original parents and the several diverse systems of breeding require for their presentation eighty-two numbered formulae.

August 25, 1915.

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HEREDITARY ANCHYLOSIS OF THE PROXIMAL PHALANGEAL JOINTS (SYMPHALANGISM)

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INTRODUCTORY

Various types of developmental malformation of the hands and feet have been described under such terms as polydactylism, syndactylism, brachydactylism, ectrodactylism, macrodactylism, hyperphalangia and hypophalangia.[1]

Familial tendencies have been recognized in the case of some of these anomalous conditions, but the first definite record showing that one of them is a unit-character inherited in accordance with Mendelian laws was made in 1905 by WM. C. FARABEE. His recorded family were residents of Pennsylvania and in the affected members there occurred a shortening of the fingers and toes which possessed only one instead of two interphalangeal articulations (hypophalangia or brachydactylism). Subsequently DRINKWATER (1908) described an English family which had exhibited the same dominant character through seven generations. The affected persons in both FARABEE'S and DRINKWATER'S families, which were possibly related, possessed a notably shortened stature, so that probably more than one abnormal unit-character was concerned.

Still another type of familial malformation had been previously described by WALKER (1901) as a particular form of anchylosis involving more especially the midphalangeal joints. The number of his recorded cases was too small to justify any conclusions on a Mendelian basis, though the malformation, the degree of which varied considerably in the different individuals, had been transmitted through five generations. In one of the cases there was an actual loss of the terminal phalanges of the two postaxial (ulnar) fingers, but otherwise there were no abnormalities aside from the aforementioned anchylosis. This paper will present a

[1] Isolated examples are not uncommon. A brother and sister were recently admitted to the Brigham Hospital, for congenital cataract. Both of them had 12 toes and brachydactylous hands. Both were of very short stature. Their parents were first cousins and were unaware of a condition similar to that of their children in earlier generations.

much more complete family record of an inherited trait of similar nature which closely accords with Mendelian expectations. The malformation will be designated *symphalangism.*

In January of 1906 the writer was consulted by a patient with a cerebral glioma, who presented in addition to her specific malady an unusual condition of the fingers which could not be bent at the proximal interphalangeal joints. An uncle who accompanied her was similarly affected and her physician who was also present, had married her fifth cousin who likewise had what they all referred to as "straight fingers" in contradistinction to the normal which they termed "crooked fingers."

In no respect was there any indication of physical deterioration in the three members of the family that were interviewed. They were splendidly developed, vigorous people. The uncle was very tall, over six feet in height, and both of the women were large and fine looking. None of them regarded the malformation as in any sense a disability or as anything out of the ordinary deserving comment. They stated that in one instance a member of the family with presumably normal fingers had married an unrelated individual with normal fingers and there were "straight fingered" children among their offspring. Aside from this possible exception (to which we will return) it was recognized in the community that if the parents were both exempt, the children would be likewise.

THE INHERITED TRAIT

Judging from the hands of the three observed individuals, though the underlying condition is the same in each, the acquired appearances justify their description under three clinical types: 1, the type of hand shown by the distant cousin who had never been accustomed to rough manual work of any kind; 2, the type in which moderate manual labor had been indulged in, as was the case with the patient, a woman who had used her hands for domestic labor; and finally, 3, the type which has undergone some modification owing to hard manual labor, as exemplified by that of the uncle, a farmer, who was accustomed to driving a plow.

These three types are fairly well represented by the accompanying photographs. In figure 1 are shown the hands of the distant cousin (Gen. V, Fam. 3, Child 7). The photograph was taken to show the unusual degree of flexion acquired in the terminal phalangeal joints which serves in a measure to counteract the incapacitation of the deformity The view shows at the same time the smooth column-like effect

FIGURE 1.—The hands of the distant cousin unmodified by manual labor, showing the column-like fingers with their unusual capacity of over-flexion of the terminal phalanges. The proximal joint absent in all fingers.

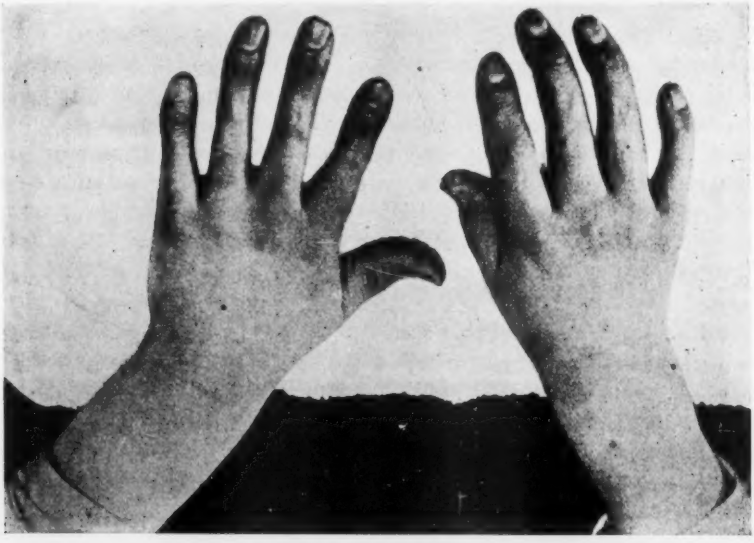


FIGURE 2.—Hands of the patient, the right modified by paralysis. No proximal interphalangeal joints present except in right index where it is slightly developed (cf. figure 7).

of the proximal two-thirds of the eight fingers with their smooth unwrinkled skin. In figure 2 are seen the hands of the patient (Gen. VI, Fam. 34, 5th Child), the right showing, at rest, slight flexion at the terminal phalanges owing to the coexistent paralysis. In the left hand it can be seen as in the above that there are no transverse cutaneous wrinkles of the proximal interphalangeal joints, the skin over the median portion of the fingers where they were thinner than normal being glossy and smooth like the skin characterizing certain neuropathic disturbances. There was a slight suggestion of flexion in the near joint of the right index finger to be explained by the X-ray findings. In figure 3 are

FIGURE 3.—The hands of the uncle as modified by hard manual labor. The indices possess a proximal interphalangeal joint but its development is imperfect in the other three fingers of each hand. (Cf. X-ray figure 9.)

shown the hands of the uncle, one of twins (Gen. V, Fam. 8, Child 7). At first glance they resemble somewhat the gnarled hands of an arthropathy but it can be seen that, except for the two indices, there is no cross wrinkling of the skin corresponding with the proximal phalangeal joints. In these two fingers slight movement was possible at these joints but in the other three the proximal joints were immovable, and though slightly bent the skin over them was smooth and unwrinkled, for no movement was present in them.

Though these "straight fingered" hands are very useful for all ordinary purposes, the succeeding three figures show how incapable these individuals are of effectively closing the hand into a fist. Figure 4 shows

FIGURE 4.—Patient's left hand (cf. figure 2) with hand almost fully flexed.

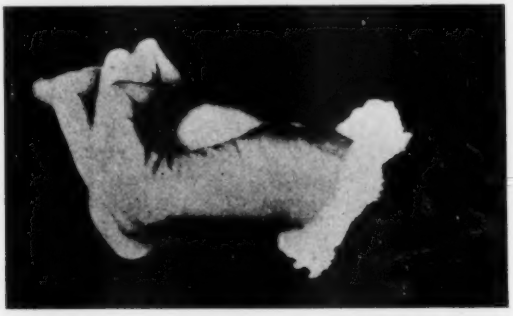


FIGURE 5.—Cousin's hand (cf. figure 1) in fist position.

the left (the intact) hand of the patient flexed to its full extent; figure 5 shows that of the cousin similarly closed; and figure 6 that of the uncle, which can be more completely closed owing to the slight permanent flexion, through toil, in the rigid joints.

PATHOLOGY OF THE LESION

The nature of the condition is disclosed by the radiogram (figure 7) of the patient's hands, which indicates a condition identical with,

or closely resembling, that which characterized WALKER's family. It would appear that some sort of an anchylosing process had occurred

FIGURE 6.—Hand of the uncle (cf. figure 3) in fist position.

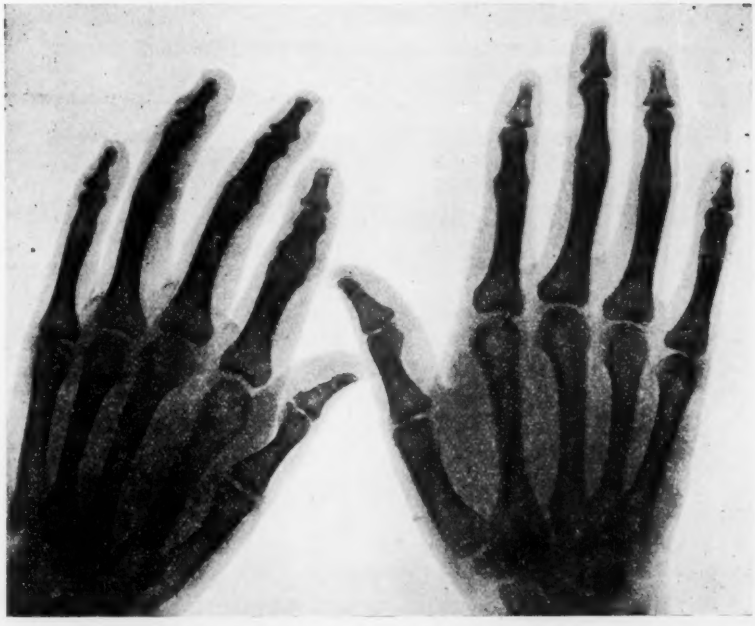
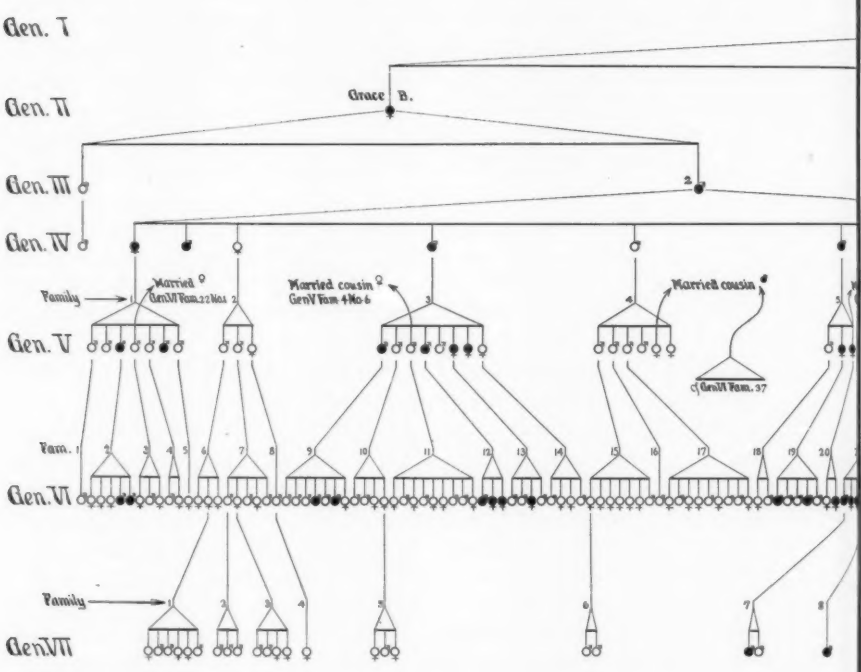
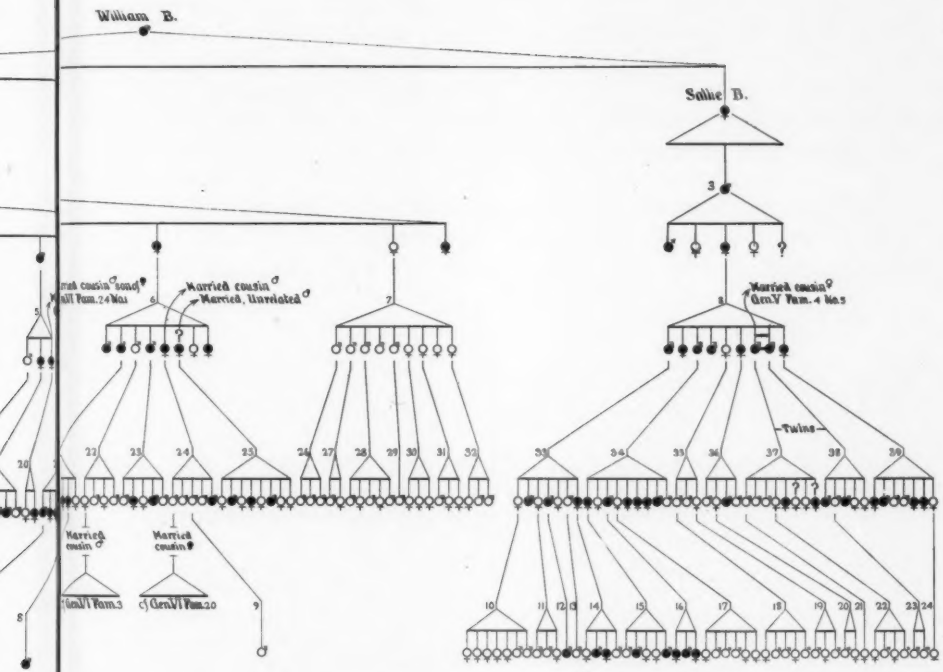


FIGURE 7.—Roentgenogram (reversed) of the patient's hands (Gen. VI, Fam. 34, No. 5), showing the anchylosis of all proximal interphalangeal joints with exception of right index.

through incomplete prenatal development of the joint, for though a trace of the joint is present with corresponding slight enlargements of the ends of the adjoining phalanges, it is largely masked by a fusion of the two bones. In the index finger of the right hand alone does a definite joint appear, though it was capable of very slight movement, and it may be added that, in hands of other members of the family from whom tracings and radiographic plates have been secured, the index finger tends to approximate the normal more often than any of the others. It is to be noted moreover that the middle phalanx of the right index finger is dwarfed, and this would suggest that possibly all of these manual deformations of congenital origin represent a very closely related condition. For it will be recalled that this dwarfing of the mid-phalangeal row was the main characteristic of DRINKWATER's cases of minor brachydactyly and it appears also in some of the radiograms of WALKER's family.

Just why the phalanges of the second row should suffer more than the others in most of these inherited malformations is difficult to understand, though doubtless it is associated in some way with the late appearance of the ossification centers in this phalangeal row. As shown by MALL (1906) and others, these are the last of the three rows of phalanges to ossify. Curiously enough, the terminal row of phalanges are the first to show centers of ossification, and apparently the centers are laid down serially from the first to the fifth finger, all of them having acquired a center of ossification by the 56th day. In the first or proximal row, the ossification centers for the index and middle finger only are present in the 58-day embryo, and not until the 64th day have centers appeared in this row for all the fingers. The second or middle row do not begin to ossify until the 75th day, when a center is present in the index finger, but the center for the fifth finger does not appear until the 83rd day. Hence it is possible that some inhibitory influence may be active at this prenatal period, which may account for these congenital anomalies, and it is conceivable that such an influence may be inherited. This would account not only for the dwarfing or non-development of the middle phalanges but also for the predominance of involvement of the postaxial (ulnar) fingers. This suggestion, however, though possibly explaining the underlying lesion characterizing brachydactyly, will not fully account for the condition with which we are dealing. For though a certain degree of brachydactylism does occasionally appear, the chief characteristic of symphalangism is a joint-lesion. Although we do not know what is the chronological succession of development for the interphalangeal


Gen. I
Gen. II
Grace B.
Gen. III
2
Gen. IV
Family
Married ♀
Gen.VI Fam.22 No.1
Married cousin ♀
Gen.V Fam. 4 No. 6
Married cousin ♂
Gen.VI Fam. 37
Gen. V
Fam.
Gen.VI
Family
Gen.VII


William B.
Sallie B.
3
Married cousin ♀
Gen.V Fam. 4 No.5
Married cousin ♂
Married, Unrelated ♂
Twins
Married cousin ♂
Married cousin ♀
Gen.VI Fam.3
Gen.VI Fam.20

joints, it is quite probable that the articulations between the proximal and middle rows of phalanges are the last to be laid down, and that in these individuals an inhibitory influence checks their development at a stage a few days later than that which checks the formation of the ossification centers and produces brachydactyly.

The stages for an individual finger therefore would appear to be (1) a complete inhibition of growth of the middle phalanx (major brachydactyly), (2) a dwarfing of the middle phalanx (minor brachydactyly), with or without joint alteration, (3) a perfect development in length of the middle phalanx, but with an imperfect or absent proximal joint (symphalangism). Owing to the fact that the time of development does not exactly coincide in the several fingers, it might be possible in a single hand to find approximate examples of the three types of developmental inhibition.

It is apparent furthermore from the X-rays accompanying the papers by WALKER and DRINKWATER (minor brachydactyly), as well as from my own studies, that the postaxial side of the hand is chiefly affected, more especially the little finger. Thus in the phalangeal fusion typical of the family on which this paper is based, grades of involvement can be illustrated as follows:

(1) Fingers of normal length (cf. figure 1) but with bony anchylosis of the proximal joints of all four fingers (cf. patient's left hand, figure 7).

(2) Fingers of normal length anchylosed as above, except in the index finger which possesses an imperfect proximal inter-phalangeal joint, as was the case with the patient (cf. right hand figure 7) as well as (cf. figure 8) of one of her sisters (Gen. VI, Fam. 34, Child 6).

(3) Fingers of approximately normal length as in the case of the twin uncles (cf. hand of twin, Gen. V, Fam. 8, Child 8, figure 3), but which show a fairly well developed joint of the index, an imperfect joint of the middle, and bony fusion of the ring and little fingers, the latter evidently having an imperfectly developed middle phalanx; in fact all the middle phalanges are somewhat short (figure 9).

(4) Fingers of the hand of a supposedly unaffected daughter (Gen. VI, Fam. 37, Child 8) of one of the affected twins, in whom the X-ray shows an imperfect proximal inter-phalangeal joint and a dwarfed middle phalanx of the little finger alone (figure 10).

Hence the rule may be established for these conditions not only that *the middle row of phalanges and the proximal phalangeal joints suffer most severely,* but that *in lesser grades of the given malformation the nearer the preaxial (radial) side of the hand, the less outspoken is the lesion.* This however must be taken with certain reservations, so far as

the index finger is concerned, though it appears to be the rule for the three others.[2]

In many, if not in all, of the individuals in the family which is the subject of this report, the toes are likewise affected, and, as will be

FIGURE 8.—Patient's sister: (Gen. VI, Fam. 34, Child 6) showing imperfect proximal interphalangeal joint of index finger without notable brachydactylous change of mid-phalanx. (Compare with right hand of figure 7.)

[2] One cannot resist the impression given by some of these radiograms, particularly by those of certain members of WALKER'S family, in whom the two terminal phalanges were actually missing from the fourth and fifth fingers (figure 11), that we may be dealing with a condition allied in some way with the elimination of the inner and outer digits in hoofed animals with five-toed ancestors.

seen, the involvement of the feet, in the case of the individual who presumably has normal hands and yet some of whose children have "stiff

FIGURE 9.—Hand of one twin brother (Gen. V, Fam. 8, Child 7) showing tendency of preaxial fingers to escape from the anchylosing process. The nearer the postaxial (ulnar) side the more severe the involvement.

fingers," stamps her as presumably involved. I am informed that there have been instances of webbed toes and of supernumerary toes, as well as of anchylosed ones, but concerning this I have no exact information.

FAMILY RECORD

The original member of the family, William B., who was known to have been the possessor of "straight fingers," migrated to this country from Scotland in 1700 and settled in Virginia. He married, had

FIGURE 10.—Hand showing the mildest degree of involvement, confined to the most postaxial finger (Gen. VI, Fam. 37, Child 8).

a large family of children, some of them with "stiff fingers," and from this source have sprung the seven generations, most of whom still remain in the neighborhood country.[3]

The community in which these people live is situated in two valleys in Tazewell County, in the southwestern part of Virginia. One of these valleys is a curiously bowl-shaped depression surrounded by mountains on all sides except for a narrow pass, and it is here that many of the descendants of William B's daughter, Sallie B., reside. Other collaterals live in a much larger adjoining valley. There are some "straight fingered" descendants in every branch, and though intermarriages have occurred, they have been comparatively few (cf. chart).

FIGURE 11.—An example of interphalangeal anchylosis with absence of the terminal phalanges of the two ulnar fingers. (From WALKER.)

[3] One of the twins (Gen. V, Fam. 8) of this American branch has had communication with a distant relative in Scotland who bears the same name as the founder of the American line and who shows the family trait.

So far as could be learned, these individuals were unrelated to the Maryland family reported by WALKER, though it is quite probable that offshoots of some earlier generation, of whom there is no record, may have migrated, for as can be seen by the chart, these people have been very prolific. William B. had several children, many of them affected, and our present record includes a line of descent from only two of them; indeed as far down as the fourth generation in the Sallie B. line, there were several children in one family whose offspring no attempt has been made to trace.[4]

The Sallie B. line of the chart which contains the tree of the patient's family was made out for me by the uncle and the details have been since corroborated from two other sources. The Grace B. line was secured by my friend, Dr. S. J. CROWE, who some years ago made a special visit to the community and secured the data on which it is based. This record has also been subsequently corrected and authenticated and further details have been added to it from time to time by my correspondent, a resident of the valley, to whose intelligent coöperation I am greatly indebted.

These two lines including the progenitor William B. comprise, all told, 313 individuals, and the following analysis may be made of them. In the *second generation,* the two recorded daughters from the large family of the original William B. were both "stiff fingered." In the *third generation,* there are only three recorded individuals, two with and one without the anomaly. In the *fourth generation,* there are fourteen individuals, eight recorded with and six without the anomaly, there being six affected and three free in the single fully recorded family.

In the *fifth generation,* the numbers increase sufficiently to be of some statistical value. Of the 53 individuals there are 18 unaffected children of 3 unaffected parents, whereas the 5 affected parents had 22 affected children and 13 unaffected children. This predominance of affected over unaffected is largely influenced by the one family (Gen. V, Fam. 8) in which there were 8 affected (2 of them twins) and but 1 unaffected child.[5] In the Grace B. line the 26 children of affected parents are about evenly divided, there being 14 affected and 12 unaffected children.

[4] My correspondent tells me that there are "stiff fingered" people, known descendants of William B., in Bland, Va., and another group of them in Princeton, West Virginia, who are presumably related.

[5] Particular inquiries have been made regarding the parents of this family and the supposedly unaffected daughter, owing to the suspicion that the parents might both have carried the trait and have been homozygous. But the father came from a distant community and was unrelated, and there can be no question but that the daughter is unaffected.

In the *sixth generation,* as the result of the recorded 39 marriages from the foregoing, there were 169 children, 79 (about 50 percent of them) being the unaffected offspring of 23 parents recorded as unaffected; whereas from the 16 affected parents there were 90 children, 40 of whom were affected and 50 unaffected.

In the *seventh generation,* as the result of 24 recorded marriages, there were 71 offspring, 55 unaffected of 18 unaffected parents, whereas from the 6 affected members of the previous generation who married, there were 10 affected and 6 unaffected children.

To analyze our data further, we can see that among the 312 descendants of William B. recorded in the chart there were 84 affected persons, a few more than the 25 percent of the total number which would be expected, did the affected and unaffected individuals mate with equal frequency and have an equal number of offspring. If however we exclude from this computation the incomplete families of the first few generations in which merely the affected individuals carrying "the lines" are given, we have 72 completed families in the four succeeding generations comprising 302 individuals, 78 of them being affected, namely 25.8 percent.

That there has been no discrimination in the mating of males or females whether affected or unaffected is apparent when we take a single generation such as Gen. V, in which all marriages likely to have fruition have presumably been made; for of the 53 individuals, there are 22 affected, 16 of whom (73 percent) have married and have families, whereas there are 31 unaffected, 23 of whom (74 percent) have married and have families. In the fourth and fifth generations an equal number, namely 8 affected and 8 unaffected individuals, have either not mated or have been without offspring.

It is obvious from the chart that there is no sex-limitation in the transmission of the trait, there being 17 affected male parents with affected offspring and 15 affected female parents with affected offspring.

To trace this matter further, in the 72 families of the later generations for which the records are complete, 44 of them were from the mating of unaffected parents with 152 children, all of course unaffected. On the other hand, in the 28 families with an affected parent there were 150 children, showing that the affected individuals have been proportionately more prolific than the unaffected. *Of the 150 children of affected parents in these 28 completed families, 78 of them, or 52 percent, carried the trait,* a figure which closely corresponds therefore with the normal Mendelian ratio for a single unit-character.

THE SUPPOSED EXCEPTION TO MENDEL'S LAW.

In the sixth generation in the line of Grace B. there occurs what was regarded in the community as the single instance (Gen. VI, Fam. 25) of affected offspring (3 out of 8 in the family) from a presumably normal parent or at least from a parent with so slight a trait that it was not evident on casual observation of the hands. Suspicions were naturally aroused in regard to this family and unsuccessful efforts have been made to secure an X-ray photograph of the mother's hands. The suspicions, however, are in a measure confirmed by my correspondent who is a relative, and who writes that "she apparently has not stiff fingers, but she walks like the stiff fingered people whose toes or ankles are probably also stiff." On this basis she has been included (though questioned) with the affected individuals. Such a mild degree of involvement as is shown in figure 10 in the hand of a girl[6] (Gen. VI, Fam. 37, Child 8), who though originally tabulated as a suspect by Dr. CROWE was regarded as unaffected by my correspondent until the X-rays were taken, indicates how cautious one must be, without an examination, in accepting data concerning individuals who may pass for normal in the community. Still on this basis it is remarkable that local tradition records no other example of a suspected recession through the mating of presumed normals.

INTERMARRIAGES

It will be seen from the chart that there have been a few intermarriages though there is no recorded marriage of two affected individuals. In Generation V an unaffected son (Fam. 1, Child 4) of an affected mother married an unaffected relative of Generation VI (Fam. 22, Child 1) with three unaffected offspring. In the same generation another unaffected son (Fam. 3, Child 3) married an unaffected first cousin (Fam. 4, Child 6) who bore nine unaffected children. In Generation V also, an affected woman (Fam. 5, Child 3) married an unaffected relative of Generation VI (Fam. 24, Child 1) and has a family of two children, one of whom is affected. A third intermarriage in Generation V occurred between one of the affected twins (Fam. 8, Child 7) and an unaffected cousin (Fam. 4, Child 5) and among their eight children (Gen. VI, Fam. 37) five are free, only one, the fifth child, unmistakably carries the trait and she has only two fingers involved; another, the eighth child, is very mildly affected, with only one finger involved (figure 10), and a third is merely a suspect.

[6] Her elder sister (Child 6), also a suspect, has not been X-rayed.

MODIFICATION OF DOMINANCE

It is of interest that there is a local tradition which would signify that the dominance of the anomaly is becoming modified, for it is believed that the children of "stiff fingered" people are apt to have one less finger involved than their parents and I judge that the index finger, as has been indicated, is the first to escape, whereas the postaxial fingers tend longest to retain the anomaly. This tradition might be borne out by the line of Sallie B., for in the single recorded family of the fifth generation, though 8 out of the 9 children were affected, many of their offspring in the next generation are but mildly affected and this is particularly true of the children of the twins, who are, I may add, identical twins, even to the degree of the anomaly, as shown in the X-rays of their hands.

Opposed to this suggestion of fading dominance, however, is the following experience in the line of Grace B., for one of her grand-daughters (Gen. IV, Fam. 2, Child 7) who was only mildly affected, having merely the two postaxial fingers on each hand involved, had a family of eight children, five of whom had extensively involved hands, and a sixth was the suspect (Gen. V, Fam. 6, Child 6) whom we have discussed and whose family of eight contained three children with extensively involved hands.[7] It is probable therefore that *the trait may be transmitted in its most outspoken form by a parent who appears to be but slightly affected, but never through parents both of whom are unaffected.*

CONCLUSION

An abnormal character, designated symphalangism, which is transmissible by either sex, appears in seven generations as a bony anchylosis of the proximal interphalangeal joints of one or more fingers. In its milder degrees, the preaxial (radial) fingers are less likely to be involved than the postaxial. Both hands and feet of the affected individuals may be affected. The trait may be transmitted in its most outspoken form by a parent in whom it is inconspicuous, but never by unaffected parents.

Of 302 individuals in 72 completely recorded families, 25.8 percent of them were affected. Of the 150 children of the affected parents, 78 of them, or 52 percent, were affected. Hence the trait behaves as a

[7] Dr. W. E. CASTLE, who has been kind enough to look over this data, suggests that other inherited factors may affect the manifestation of the character without necessarily affecting its dominance, so that it may crop out in full force in later generations even if partially inhibited by these other factors in a particular case.

simple Mendelian dominant with an equal chance, among the offspring of affected individuals, that it will be or will not be inherited.[8]

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[8] Since this article was submitted for publication, additional data have been received concerning 139 other individuals, including the West Virginia branch of the family, reaching into eight generations. Thus, all told, the tabulations comprise 452 individuals, though the percentages of affected and unaffected remain practically as computed from the two lines here presented. Moreover an X-ray photograph has been finally secured, unfortunately of the hand alone, of the suspect who was fully discussed, and it shows a brachydactylous median phalanx of the little finger of one hand.